FLEXURE OF ORTHOTROPIC ROTATING DISKS

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Summary—The influence of off-set in mounting the blades on the disk, termed "eccentricity", and the centrifugal stiffening on the stresses and deflection induced in laterally loaded orthotropic disks of variable thickness is analysed. The analysis is based on a series solution for the differential equation governing the deflection of the disk. Numerical results showing the effects of anisotropy, eccentricity and rotation on the stresses and deflection of the disk are presented graphically. It is shown that the stress due to radial bending moment reduces significantly with the increase in the degree of anisotropy.

NOTATIONS

- $a_1$: distance between the outer edge of the disk and the centre of gravity of the blades
- $a_2$: distance between the outer edge of the disk and the point of application of steam (gas) force
- $D_1$: $\alpha - \mu_\theta$
- $D_2$: $\beta - \mu_\theta$
- $D_3$: $\mu_\theta - \lambda^2$
- $E_1, E_2$: elastic moduli in 'r' and '0' directions respectively
- $e$: departure of the centre of gravity of the blade from the middle plane of the disk (Fig. 1)
- $h, h_0$: local thickness and thickness at radius $r_o$ respectively
- $K_1$: steam (gas) force on the blade
- $K_2$: force function
- $K_3$: force function
- $K_4$: force function
- $m$: $(3 + \mu_\theta)
- n$: thickness parameter in $h = h_x^x$
- $p$: unidirectional axial pressure of the steam (gas) acting on the face of the disk
- $r_o$: radius of curvature of the deflected middle surface of the disk
- $r_i, r_o$: inner and outer radii of the disk
- $u, z$: radial displacement and axial displacement of the middle plane
- $W_o$: weight of the blades
- $\alpha, \beta$: $-n \pm \sqrt{\left(\frac{n^2}{4} + \mu_\theta n + \lambda^2\right)}$
- $\delta$: $(\mu_\theta - \lambda^2)$
- $\lambda^2$: degree of anisotropy, $E_1/E_2$
- $\mu_\theta$: Poisson's ratio characterising compression in 'r' direction due to tension in '0' direction
- $\rho$: weight per unit volume of the disk
- $\sigma_1, \sigma_2$: stresses, due to rotation only, in radial and circumferential directions respectively
- $\sigma_3$: stresses, due to rotation only, in axial direction
- $\sigma_4$: $K_1x^{n+1} + K_2x^{m+1} + ax^2$
- $\sigma_5$: $K_1x^{n+1} + K_2x^{m+1} + ax^2$
- $\sigma_6$: maximum bending stresses in radial and tangential directions respectively
- $\sigma_7$: total stress, $(\sigma_1 + \sigma_2^R, (\sigma_1 + \sigma_2^T))$, in radial and tangential directions respectively
- $\tau_m$: mean axial shear stress in ring section of the disk at a radius "r"
- $\omega$: angular velocity of the disk

INTRODUCTION

The bladed disk is a key unit of turbo-machinery and a successful design of the disk necessitates an analysis of stresses and deflection that takes into account the entire force system acting on the "disk-blade" assembly. With the increased use of aniso-

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tropic and laminated composite materials in the fabrication of machine components and structural units, it has become imperative to consider the anisotropic characteristics of the material.

In this paper, stresses and deflection induced in a rotating orthotropic disk of variable thickness due to a force system comprising
1. the axial pressure of the steam (gas) on the disk,
2. the steam (gas) force on the blade,
3. the centrifugal force due to rotation,
4. the weight of the blade and
5. the weight of the disk
are obtained. Besides these, the effect of "eccentricity" which is related to the setting of the blades on the disk, and the stiffening effect due to rotation are also considered. The resulting differential equation governing the deflection has been solved and the numerical results evaluated for various values of the degree of anisotropy and the eccentricity, are represented graphically.

It may be noted that the elastic stresses in an orthotropic disk due to rotation alone and stresses and deflection in a stationary disk due to lateral pressure alone have been studied [1, 2]. The isotropic version of the present problem is known [3].

ANALYSIS

Fig. 1 shows the disk geometry and loading. The analysis is carried out within the framework of linear theory of elasticity and assumes plane stress state. In addition, it is assumed that the principal axes of anisotropy coincide with the \( r, \theta, z \) directions.
The stresses at any point on the surface of a rotating disk subjected to symmetrical lateral loading are given by

\[
\sigma_r = \sigma_r^T + \sigma_r^s, \\
\sigma_\theta = \sigma_\theta^T + \sigma_\theta^s.
\] (1)

\[
\sigma_r^T = \sigma_r + \sigma_r^b.
\] (2)

### Stresses due to rotation

The force function \( F \), defined as \( \sigma_r = F \rho \) and \( \sigma_\theta = \frac{1}{r} \frac{dF}{dr} + \frac{\rho}{g} \omega^2 r^2 \), which satisfies the equation of equilibrium and compatibility condition for a variable thickness disk can be obtained from the equation [1]

\[
r^2 \frac{d^2 F}{dr^2} + (1 + n) r \frac{dF}{dr} - (3 + n) \omega^2 r^2 F + \frac{3 + n}{2} \omega^2 r^2 h_n = 0
\] (3a)

whose general solution is

\[
F = A \rho + B \rho \omega^2 r^2 - n
\] (3b)

where \( \alpha \) and \( \beta \) are the roots of the characteristic equation resulting from the homogeneous part of equation (3a). The stresses and the radial displacement are

\[
\sigma_r = \frac{A \alpha^{n+1} + B \beta^{n+1}}{h_n e_a} + \frac{\rho}{g} \omega^2 r^2
\] (4)

\[
\sigma_\theta = \frac{A \alpha^{n+1} + B \beta^{n+1}}{h_n e_a} + (m(3 - n) + 1) \frac{\rho}{g} \omega^2 r^2
\] (5)

\[
u = \frac{1}{E} \left( \frac{A \alpha^{n+1} + B \beta^{n+1}}{h_n e_a} + \frac{\rho}{g} \omega^2 r^2 \right)
\] (6)

For a turbine disk with radius \( r_n \) mounted on a rigid circular shaft of radius \( r_0 \), the boundary conditions are:

\[
u \mid_{r=r_0} = 0,
\sigma_r \mid_{r=r_0} = \frac{W_o \omega^2 (r_0 + a_i)}{2 \pi r_0 h_n g}
\] (7)

which lead to the constants of integration, \( A_i \) and \( B_i \), as

\[
A_i = \frac{\rho h_n \omega^2 r_n^2}{g r_n^4} [K_i]
\] (8)

\[
B_i = \frac{\rho h_n \omega^2 r_n^2}{g r_n^4} [K_i].
\] (9)

Substituting for \( A_i \) and \( B_i \) in the equations (4) and (5) we have

\[
\sigma_r = \frac{\rho}{g} \omega^2 r_n^2 [\sigma_r^T]
\] (10)

\[
\sigma_\theta = \frac{\rho}{g} \omega^2 r_n^2 [\sigma_\theta^T].
\] (11)

### Stresses due to bending

Since the maximum total stress is tensile in nature attention is focussed on the tensile fibre. Referring to Fig. 1, we have:

1. The centrifugal force experienced by the disk edge due to the blade is

\[
F_c = \frac{W_o}{g} \omega^2 (r_n + a_i).
\] (12)

2. The axial shear stress at any section is

\[
\tau_m = \frac{[K_c + W_o + \pi (r_n^2 - r^2) \rho + 2 \pi \rho \int_{r_0}^{r_n} rh \, dr]}{2 \pi rh}.
\] (13)

The equilibrium equation is [3]

\[
\frac{d}{dr} (rh^2 \sigma_r) - \sigma_\theta h^2 = 6hr \left( \sigma_r \frac{dz}{dr} - \tau_m \right).
\] (14)

Due to bending, the strain in the radial direction experienced by the outer fibre of the circular section of radius \( r_n \) is

\[
e_r = \frac{h}{2 R_e}
\] (15)
which reduces to $h\zeta''$ since, for small deflection $\frac{1}{E_o} \frac{dz}{dr^2} = \zeta''$. The strain induced in the tangential direction, due to a change in the radius of the circle from $r$ to $r + \frac{h\zeta'}{2}$, because of bending, is given by

$$e^b_\theta = \frac{h\zeta'}{2r}. \tag{16}$$

The stress-strain relations (with the Maxwell relation, i.e. $\frac{E_\theta}{E_\theta} = \frac{\mu_\phi}{\mu_\rho} = \lambda$) are

$$e^b_\theta^r = \frac{1}{E_o} \left[ \lambda \sigma^r - \mu_\rho \sigma^\theta \right] \tag{17}$$

$$e^b_\theta^\theta = \frac{1}{E_o} \left[ \sigma^\theta - \mu_\rho \sigma^r \right]. \tag{18}$$

Substituting (15) and (16) in (17) and (18) respectively, and simplifying for $\sigma^r$ and $\sigma^\theta$ we get

$$\sigma^r = \frac{h}{2} \left( \frac{E_o}{\lambda^2 - \mu_\rho^2} \right) \left( z'' + \frac{z'}{r} \right) \tag{19}$$

$$\sigma^\theta = \frac{h}{2} \left( \frac{E_o}{\lambda^2 - \mu_\rho^2} \right) \left( z'' + \frac{z'}{r} \lambda \right) \tag{20}$$

Utilising (19) and (20) in (14) we have the differential equation governing the deflection as

$$r^2 \zeta''' + r \zeta'' \left( 1 + 3 \frac{h'}{R} \right) - \zeta' \left( \lambda^2 - 3 \mu_\rho \frac{h'}{r} \right) = 12 \frac{r'}{h} \left( \lambda^2 - \mu_\rho^2 \right) \left( \sigma_\theta z'' - \tau_n \right). \tag{21}$$

It may be noted that equation (21) reduces to that of the isotropic case[3], when $\lambda^2 = 1$. For a variable thickness disk whose thickness varies as $h = h_0 r^n$, equation (21) becomes

$$r^2 \zeta''' + r \zeta'' \left( 1 - 3n \right) - \zeta' \left( \lambda^2 - 3 \mu_\rho \frac{h'}{r} \right) = 12 \frac{r'}{h_0} \left( \lambda^2 - \mu_\rho^2 \right) \left( \sigma_\theta z'' - \tau_n \right). \tag{22}$$

It may be noted that equation (22) reduces to that given in reference (2) by setting $\omega = 0$. The term $\sigma_\theta$ represents the effect of the stiffening of the disk due to rotation and renders equation (22) intractable for closed-form solution. Hence, a series solution on the lines followed in reference (3) is attempted.

The equation (22) is recast in the form

$$r^2 \zeta''' + p(r) \zeta'' + q(r) \zeta' + s(r) = 0 \tag{23}$$

and with

$$z(x) = \frac{Z(r)}{r_n}, \quad z'(x) = \frac{dz(x)}{dx} = \frac{dz(r)}{dr} = z'(r) \quad z''(x) = r_n z''(r), \quad z'''(x) = r_n^2 z'''(r)$$

and equation (23) reduces to

$$X^2 z''(x) + p(x) z'(x) + q(x) z(x) + S(x) = 0 \tag{24}$$

where

$$p(x) = x (1 - 3n),$$

$$q(x) = -\lambda^2 - 3 \mu_\rho \frac{h'}{r} - 12 \lambda^2 \mu_\rho \left( \frac{\sigma^\theta}{E_o} \right) \left( \frac{f_n}{h_0} \right) \lambda^n z'' + 12 \left( \lambda^2 - \mu_\rho^2 \right) \frac{r'}{h_0} \left( \frac{f_n}{h_0} \right) \lambda^n z'';$$

$$S(x) = +12 \left( \lambda^2 - \mu_\rho^2 \right) \frac{r'}{h_0} \left( \frac{f_n}{h_0} \right) \lambda^n z''.$$}

The deflection of the disk fixed at the bore, can be approximated by[3]

$$z(x) = A(x - x_0)^2 \left( 1 + A_1 (x - x_0) + A_2 (x - x_0)^2 + A_3 (x - x_0)^3 \right) \tag{25}$$

where $A$ is the constant of integration defined by the boundary condition

$$z'(x_0) + z'(x_0) \mu_\rho = 2 \left( \frac{f_n}{h_0} \right) \left( \lambda^2 - \mu_\rho \right) \left( \frac{\sigma^\theta}{E_o} \right), \text{ for } x = x_0 \tag{26}$$

derived from equation (19). The constants $A_1$, $A_2$ and $A_3$ are calculated by introducing (25) in (24) with the stipulation that the equation shall be satisfied at three stations, $x = x_0$, $x = x_0 + 1$, and $x = x_0 - 1$. Examination of equation (25) and its derivatives indicate that at $x = x_0$ (inside the boundary of the disk)
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\[ z'(x_0) = 0, \quad z''(x_0) = 0, \quad z''(x_0) = 2A, \quad \text{and} \quad z^{(n)}(x_0) = 6AA_n. \]
Hence, for \( x = x_0 \), equation (24) reduces to
\[
AA_1 = \frac{S(x_0) - p(x_0)A}{6x_0^2} - \frac{p(x_0)A}{3x_0^2}.
\]

Similarly, equation (24) with (27) reduces to
\[
AA_2P_1 + AA_2Q_1 = AR_1 + S_1 \quad \text{for} \quad x = x_1,
AA_2P_2 + AA_2Q_2 = AR_2 + S_2 \quad \text{for} \quad x = x_2 = 1
\]

where
\[
P = 4(x - x_0)(6x^2(3p(x)(x - x_0) + q(x)(x - x_0)^3) ,
Q = 5(x - x_0)(12x^2 + 4p(x)(x - x_0) + q(x)(x - x_0)^3) ,
R = \frac{P(x_0)}{x_0^2}(2x^2 + 2p(x)(x - x_0) + q(x)(x - x_0)^3)]
- 2p(x) - 2q(x)(x - x_0) - \frac{s(x_0)}{2x_0^2}(2x^2 + 2p(x)(x - x_0) + q(x)(x - x_0)^3) - s(x).
\]

Hence, for \( x = x_0 \), equation (24) reduces to
\[
S(x_0) = \frac{p(x_0)A}{6x_0^2} - \frac{p(x_0)A}{3x_0^2}.
\]

Similarly, equation (24) with (27) reduces to
\[
AA_2P_1 + AA_2Q_1 = AR_1 + S_1 \quad \text{for} \quad x = x_1,
AA_2P_2 + AA_2Q_2 = AR_2 + S_2 \quad \text{for} \quad x = x_2 = 1
\]

With the further notation
\[
B_1 = \frac{RQ_1 - RQ_2}{PQ_1 - PQ_2}, \quad B_2 = \frac{S_1Q_1 - S_2Q_1}{PQ_1 - PQ_2},
C_1 = \frac{R_1P_1 - R_2P_1}{PQ_1 - PQ_1},
C_2 = \frac{S_1P_1 - S_2P_1}{PQ_1 - PQ_2},
\]
we obtain \( AA_2 = AB_2 + B_3, \quad AA_3 = AC_2 + C_3 \) and hence the solution
\[
z(x) = A(x - x_0)^2 \left[ 1 - \frac{p(x_0)}{3x_0^2} (x - x_0)^3 + B_3(x - x_0)^3 + C_2(x - x_0)^3 \right]
+ \left( x - x_0 \right) \left[ - \frac{s(x_0)}{2x_0^2} + B_3(x - x_0) + C_3(x - x_0)^2 \right].
\]

The value of the constant "A" is obtained from the boundary condition (26) as
\[
A = \frac{2(\lambda^2 - \mu \lambda \mu')}{2(1 + \mu \mu') (x - x_0)^2 + \mu_\lambda R'(2 + \mu_\lambda R') + 4B_2R'(3 + \mu_\lambda R') + 5C_2R'(4 + \mu_\lambda R')}
\]

where
\[
R^* = (1 - x_0)
\]

Hence
\[
\sigma^*_h = \frac{1}{2} \left( h_s \right) \left( \frac{E_s}{h_s} \right)^x x^2 \left[ A \left( \frac{2(1 + \frac{x - x_0}{x} \mu_\lambda) - \frac{p(x_0)}{x_0^2} (x - x_0)^2 + \frac{p(x_0)}{x_0} (x - x_0) \mu_\lambda}{x} \right)
+ 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]
+ \left\{ \frac{s(x_0)}{2x_0^2} (x - x_0)^2 \left( 2 + \frac{x - x_0}{x} \mu_\lambda \right) + 4B_3(x - x_0)^2 \left( 3 + \frac{x - x_0}{x} \mu_\lambda \right) + 5C_2(x - x_0)^2 \left( 4 + \frac{x - x_0}{x} \mu_\lambda \right) \right\} \right]

The above equations are general and can be applied for any type of turbine disk, with necessary
modifications, as, for example:

**Case 1.** For an impulse turbine, with disk axis horizontal and the blades mounted without any eccentricity, (weight of the disk and the weight of the blades will not induce bending moments and shear)

\[
\tau_m = \frac{K_d}{2\pi rh}, \quad \sigma^b = \frac{3(K_d a_d)}{\pi r h}
\]

The above equations are to be used in place of (13) and (30).

**Case 2.** For an impulse turbine with the axis horizontal and the blades eccentrically mounted,

\[
\sigma^b = \frac{3}{\pi r h^3} \left( K_d a_d - \frac{W_d e}{g} (r_e + a) \omega^2 \right)
\]

**Case 3.** For a reaction turbine with the axis horizontal

\[
\tau_m = \frac{(K_d + \pi (r_e^2 - r^2)) p}{2\pi rh},
\]

\[
\sigma^b = \frac{3}{\pi r h^3} \left( K_d a_d - \frac{W_d e}{g} (r_e + a) \omega^2 \right)
\]

**Numerical example**

To illustrate the effect of degree of anisotropy and eccentricity "e" on stresses and deflection, the geometry of, and the force system on, the disk are taken as follows [3]:

- \(r_0 = 0.1 \text{ m (4 in.)}\), \(r_e = 0.51 \text{ m (20 in.)}\), \(h = 0.0254 \text{ m (1 in.)}\), \(n = 0.5\), \(\omega = 3000 \text{ rpm}\), \(a = 0.3\), \(a_e = 0.051 \text{ m (2 in.)}\), \(W_d = 400.3 \text{ N (901 lbs)}\), \(K_d = 667.2 \text{ N (150 lbs)}\), \(p = 1.034 \times 10^7 \text{ N/m}^2 (15 \text{ psi})\), \(\rho = 7.84 \times 10^4 \text{ N/m}^3 (0.289 \text{ lbs/in.}^3)\), \(E = 21.25 \times 10^8 \text{ N/m}^2 (30.8 \times 10^6 \text{ psi})\).

Fig. 2 shows the stress distribution for the above disk with blade axis coinciding with disk middle plane.
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FIG. 3. Stress distribution.

FIG. 4. Deflection distribution.

FIG. 5. Maximum deflection vs $\lambda^2$. 
(\(e = 0\)) for various values of the degree of anisotropy (\(\lambda^2\)). Figs. 3 and 4 show the stress and deflection distribution with the blades mounted eccentrically (\(e = 2.5 \times 10^{-4} \text{ m}\)). Fig. 5 shows the variation of maximum deflection with the degree of anisotropy (\(\lambda^2\)) for various values of eccentricity. Fig. 6 shows the effect of the degree of anisotropy and the eccentricity on the maximum bending stresses. Fig. 7 shows the stiffening effect due to rotation for \(\lambda^2 = 2\) and \(e = -2.5 \times 10^{-4} \text{ m}\).

**DISCUSSION AND CONCLUSIONS**

The present analysis provides a method for estimating the stresses and deflection in a turbo-machinery disk. The force system due to fluid forces and rotation along with the effects of the off-set of the mounting of blades are considered. It may be noted that the radial bending stress, which is predominant, reduces significantly with increasing degree of anisotropy. For the specific problem considered, the maximum deflection increases with \(\lambda^2\) until \(\lambda^2\) reaches 2.5 (approx.) and then after it decreases with further increase in the degree of anisotropy. Also the effect of eccentricity on the deflection is larger for an orthotropic disk than for an otherwise identical isotropic disk. As can be expected, the stiffening effect on the disk due to rotation is to reduce the deflection.

**REFERENCES**