Numerical investigation of electrohydrodynamic instability in a vertical porous layer

B.M. Shankar, a, Jai Kumar, b, I.S. Shivakumara c

a Department of Mathematics, PES University, Bangalore 560085, India
b ISRO Satellite Centre, Bangalore 560017, India
c Department of Mathematics, Bangalore University, Bangalore 560001, India

A R T I C L E   I N F O

Keywords:
Electrohydrodynamic instability
Vertical porous layer
Chebyshev collocation method

A B S T R A C T

The electrohydrodynamic instability of a vertical dielectric fluid saturated Brinkman porous layer whose vertical walls are maintained at different temperatures is considered. An external AC electric field is applied across the vertical porous layer to induce an unstably stratified electrical body force. The stability eigenvalue equation is solved numerically using the Chebyshev collocation method. The presence of inertia is found to instill instability on the system and the value of modified Darcy–Prandtl number Pr D at which the transition from stationary to travelling-wave mode takes place is independent of the AC electric field but increases considerably with an increase in the value of Darcy number D a. The presence of AC electric field promotes instability but its effect is found to be only marginal. Although the flow is stabilizing against stationary disturbances with increasing D a, its effect is noted to be dual in nature if the instability is via travelling-wave mode. The streamlines and isotherms for various values of physical parameters at their critical state are presented and analyzed. Besides, energy norm at the critical state is also computed and found that the disturbance kinetic energy due to surface drag, viscous force and dielectrophoretic force have no significant effect on the stability of fluid flow.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

Natural convection in a layer of fluid saturated porous medium has constituted a pole of attraction to researchers because of its applications in several fields such as oil recovery, soil mechanics, rheology, metal casting, ceramic engineering, solid-matrix compact heat exchangers, spread of contaminants in the environment and in various processes in the chemical and materials industry, and the technologies of paper, textiles, insulating materials. Available works on this topic are concerned mostly with the study of natural convection in horizontal porous layers heated from below. The state of the art has been summarized in the books by Nield and Bejan [1] and Straughan [2,3].

The stability of natural convection in a vertical porous layer has been investigated by many authors. Gill [4] was the first to investigate the stability properties of natural convection in a vertical layer of Darcy porous medium and his findings established a rigorous ground for the use of insulating porous materials in buildings, instead of air gaps. Kwok and Chen [5] discussed different qualitative behaviors of the Darcy flow model used by Gill [4] considering the full Darcy–Brinkman model with advective inertia and also performed experiments. Qin and Kaloni [6] used energy methods to give
sufficient conditions for the stability of convection in a vertical porous slab. Most of the developments are covered in the book by de Lemos [7]. Barletta [8] reconsidered Gill’s problem and showed that the change of velocity boundary conditions from impermeable to permeable causes instability while Barletta [9] studied two-dimensional stationary mixed convection in a vertical porous layer. Rees [10] and Scott and Straughan [11] presented a new perspective on Gill’s problem by taking into account of the local thermal nonequilibrium effect, while Shankar and Shivakumara [12,13] extended Gill’s and Rees’s problems for an Oldroyd-B type of viscoelastic fluid, respectively. Recently, the effect of inertia on the stability of
buoyancy-driven parallel shear flow in a vertical layer of isotropic and anisotropic porous medium using the Lapwood–Brinkman model is discussed by Shankar et al. [14,15].

The interaction between electric and/or magnetic field on fluid flow in a porous medium is of interest in many practical applications and is important particularly in the solidification process and geophysical context. Thermal convection in the presence of a uniform applied vertical magnetic field, known as magnetocovection, has been investigated in a horizontal layer of porous medium in the past (Rudraiah [16], Alchaar et al. [17], Bergman et al. [18], Muddamallappa et al. [19], Bhatta et al. [20], Riahi [21]). The linear stability of plane-Poiseuille flow at high Reynolds numbers and in the presence of a transverse magnetic field is investigated theoretically using the multidike asymptotic approach by Makinde [22], while Makinde and Mhone [23] investigated the temporal stability of magnetohydrodynamic Jeffery–Hamel flows at very small magnetic Reynolds number. The Chebyshev spectral collocation method is employed to investigate the temporal development of small disturbances in a channel filled with a saturated porous medium under in the influence of magnetic field was studied by Makinde and Mhone [24]. Shankar et al. [25] examined magnetohydrodynamic stability of natural convection in a vertical porous slab. Electrohydrodynamics, coupling of the electric field and fluid motion, has led to many complex and interesting instability phenomena. Incipient interest in theoretical studies was limited to electrohydrodynamic(EHD) convection in a layer of dielectric fluid saturated porous medium caused by the dielectrophoretic force due to the variation in the dielectric constant with the non-homogeneous temperature gradient in the bulk flow (Moreno et al. [26], del Rio and Whitaker [27], Rudraiah and Gayathri [28], Shivakumara et al. [29,30]). The effect of a uniform horizontal AC electric field on the stability of natural convection in a vertical dielectric fluid layer was examined by Takashima and Hambata [31] while the effect of a transverse uniform magnetic field on the stability of natural convection in a vertical layer of an electrically conducting fluid was analyzed by Takashima [32].

Nonetheless, studies pertaining to the stability of natural convection in a vertical dielectric fluid saturated porous layer under the influence of electric field have been almost completely neglected. Electrohydrodynamic coupled heat transfer in a fluid saturated vertical porous layer becomes important in geophysics to study the Earth's core and to understand the performance of petroleum reservoir. Moreover, the drying process in porous media is a rather complicated process as coupled heat and mass transport phenomena are involved simultaneously. Therefore, new techniques are being used to make the drying processes more efficient and one of the effective ways to improve the overall drying kinetics is to apply an electric field (Yabe et al. [33], Lai and Lai [34]). The foregoing observations provide the motivation to investigate the stability of natural convection in a vertical layer of dielectric fluid saturated Brinkman porous medium in the presence of a uniform horizontal AC electric field.

2. Mathematical formulation

The physical configuration of the problem is illustrated in Fig. 1. We consider a dielectric fluid saturated vertical layer of a porous medium confined between two parallel vertical plates at $x=\pm h$, subject to a uniform AC electric field applied across the vertical porous layer. The plate at $x=-h$ is maintained at fixed temperature $T_1$ and fixed electric potential $V_1(=0)$, while the plate at $x=h$ is maintained at fixed temperature $T_2(>T_1)$ and at an alternating (60 Hz) potential whose root-mean-square value is $V_2$. A Cartesian coordinate system ($x, y, z$) is chosen with the origin in the middle of the vertical porous layer, where the $x$-axis is taken perpendicular to the plates and the $z$-axis is vertically upwards, opposite in the direction of gravity. The relevant basic equations under the Oberbeck–Boussinesq approximation are ([1,31]):

$$\nabla \cdot \mathbf{q} = 0$$

$$\rho_0 \left[ \frac{1}{\varphi_p} \frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varphi_p^2} (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p - \frac{\mu}{K} \mathbf{q} + \mu_e \nabla^2 \mathbf{q} + \mathbf{f}_e + \rho \mathbf{g}$$

$$\chi \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa \nabla^2 T$$

$$\rho = \rho_0 [1 - \alpha (T - T_0)]$$

where $\mathbf{q} = (u,v,w)$ is the velocity vector, $p$ is the pressure, $\rho$ is the fluid density, $g$ is the acceleration due to gravity, $\mu$ is the dynamic viscosity, $\mu_e$ the effective viscosity, $K$ is the permeability, $T$ is the temperature, $\varphi_p$ is the porosity of the porous medium, $\chi$ is the ratio of heat capacities, $\kappa$ is the thermal diffusivity, $\alpha$ is the thermal expansion coefficient, $\rho_0$ is the density at reference temperature $T=T_0=(T_1+T_2)/2$ and $\mathbf{f}_e$ is a body force term which represents the response of the fluid to the electric field and can be expressed as (Landau and Lifshitz [35])

$$\mathbf{f}_e = \rho_e \mathbf{E} - \frac{1}{2} (\mathbf{E} \cdot \mathbf{E}) \mathbf{E} + \frac{1}{2} \nabla \left( \frac{\partial \mathbf{E}}{\partial \rho} \mathbf{E} \cdot \mathbf{E} \right)$$

Here $\mathbf{E}$ is the root-mean-square value of the electric field, $\rho_e$ is the free charge density and $\varepsilon$ is the dielectric constant.

The electrical force $\mathbf{f}_e$ will have no effect on the bulk of the dielectric fluid if the dielectric constant $\varepsilon$ and the electrical conductivity $\sigma$ are homogeneous. Since $\varepsilon$ and $\sigma$ are functions of temperature, a temperature gradient applied to a dielectric
fluid produces a gradient in $\varepsilon$ and $\sigma$. The application of DC electric field then results in the accumulation of free charge in the liquid. The free charge increases exponentially in time with a time constant $\varepsilon/\sigma$, which is known as the electrical relaxation time. If an AC electric field is applied at a frequency much higher than the reciprocal of the electrical relaxation time, the free charge does not have time to accumulate. Moreover, the electrical relaxation time of most dielectric liquids appear to be sufficiently long to prevent the buildup of free charge at standard power line frequencies. At the same time, dielectric loss at these frequencies is so low that it makes no significant contribution to the temperature field. The Coulomb force term $\rho_0 \vec{E}$ in Eq. (5) is the force per unit volume on a medium containing free electric charge of density $\rho_0$. It is the strongest EHD force term and usually dominates when DC electric fields are present in dielectric fluids. The second term in Eq. (5), called dielectrophoretic force term, is due to the force exerted on a dielectric fluid by a nonuniform electric field. It is usually weaker than the free charge force term and only dominates when the AC electric field is imposed on a dielectric fluid. Therefore, the Coulomb force term has been neglected in Eq. (5) and only the dielectrophoretic force term is retained in Eq. (5). It is seen that the dielectrophoretic force term depends on $(\vec{E} \cdot \vec{E})$ rather than $\vec{E}$. Since the variation of $\vec{E}$ is very rapid, the root-mean-square value of $\vec{E}$ is used as the effective value in determining fluid motion. In other words, one can treat the AC electric field as the DC electric field whose strength is equal to the root-mean-square value of the AC electric field (Takashima and Aldridge [36]). The last term in Eq. (5) is called the electrostriction term. This term can be conveniently clubbed with the pressure in Eq. (5) and, because pressure amounts to an extra variable in incompressible flow, seems not to have any influence on the hydrodynamics.

Since there is no free charge, the relevant Maxwell equations are

$$\nabla \times \vec{E} = 0, \quad \nabla \cdot (\varepsilon \vec{E}) = 0. \quad (6a,b)$$

In view of Eq. (6a), $\vec{E}$ can be expressed as

$$\vec{E} = -\nabla V \quad (7)$$

where $V$ is the root-mean-square value of the electric potential. The dielectric constant is assumed to be a linear function of temperature in the form

$$\varepsilon = \varepsilon_0 [1 - \gamma (T - T_0)] \quad (8)$$

where $\gamma (> 0)$ is the thermal expansion coefficient of dielectric constant and is assumed to be small. For example, for 10 cs Silicone oil $\gamma = 2.86 \times 10^{-3} K^{-1}$ and $\varepsilon = 2.6 \times 10^{-11} Fm^{-1}$.
3. Basic state

It is assumed that the basic steady-state flow is laminar and fully developed. Thus,

\[ \tilde{q} = W_b(x) \hat{k}, \quad P = P_b(x, z), \quad E = \tilde{E}_b(x) \hat{i}, \quad T = T_b(x), \quad \rho = \rho_b(x), \quad \varepsilon = \varepsilon_b(x), \quad V = V_b(x) \]

(9)

where the subscript \( b \) denotes the basic state. Under this circumstance, the basic state solution is found to be (Takashima and Hamabata [31])

\[ W_b = \frac{\alpha g \beta K}{2v} \left[ x - h \cosh \left( \frac{\mu h^2}{\mu e K} \right) \sinh \left( x \frac{\mu h^2}{\mu e K} \right) \right] \]

(10)

\[ P_b = \text{const} - \rho_0 g z + \frac{\varepsilon_0 E_0^2}{2(1 - \gamma \beta x/2)} \]

(11)

\[ T_b - T_0 = \beta x/2 \]

(12)

\[ \rho_b = \rho_0 \left( 1 - \frac{\alpha \beta x}{2} \right) \]

(13)

\[ \varepsilon_b = \varepsilon_0 \left( 1 - \frac{\gamma \beta x}{2} \right) \]

(14)

\[ \tilde{E}_b = \frac{E_0}{1 - \gamma \beta x/2} \]

(15)

and hence

\[ V_b = \frac{2E_0}{\gamma B} \log \left( \frac{1 - \gamma \beta x/2}{1 + \gamma \beta h/2} \right) \]

(16)

where

\[ P = p - 0.5 \rho (\partial \varepsilon / \partial \rho)(\tilde{E} \cdot \tilde{E}) \]

(17)

is the modified pressure and

\[ E_0 = \frac{\gamma \beta V_0^2}{2} \log \left( \frac{1 + \gamma \beta h/2}{1 - \gamma \beta h/2} \right) \]

(18)

is the root-mean-square value of the electric field at \( x=0 \) and \( \beta = (T_2 - T_1)/h \) is the temperature gradient in the horizontal direction. Here, \( W_b \) and \( P_b \) have been determined under the condition that the total flux of flow across a horizontal plane \( z=\text{constant} \) is zero. It is observed that in the initial steady state the electric field does not affect the flow field.

4. Linear stability analysis

To study the linear stability of fluid flow, we superimpose an infinitesimal disturbance on the base flow in the form

\[ \tilde{q} = \bar{q} + q', \quad P = P_b(x, z) + P', \quad V = V_b + V', \quad T = T_b + T', \quad \rho = \rho_b + \rho', \quad \varepsilon = \varepsilon_b + \varepsilon'. \]

(19)

Substituting Eq. (19) into Eqs. (1)-(3), linearizing by neglecting the product of perturbed quantities, we get

\[ \nabla \cdot \tilde{q} = 0 \]

(20)

\[ \frac{1}{\varphi_b} \frac{\partial \tilde{q}'}{\partial t} + \frac{1}{\varphi_b'} \left\{ (\tilde{q} \cdot \nabla) q' + (q' \cdot \nabla) \tilde{q}_b \right\} = -\frac{1}{\rho_b} \nabla P' - \frac{1}{\rho_b} \left( \frac{\mu}{K} - \mu e \nabla^2 \right) q' + \alpha T' g \hat{k} - \frac{\varepsilon_0 E_0 \beta}{2\rho_b} \frac{\partial V'}{\partial x} \hat{i} - \frac{E_0^2 \gamma^2 \beta \varepsilon_0 E_0}{2\rho_b} T' \hat{i} \]

(21)

\[ \chi \frac{\partial T'}{\partial t} + (\tilde{q}_b \cdot \nabla) q' + (q' \cdot \nabla) T_b = \kappa \nabla^2 T' \]

(22)

\[ \gamma E_0 \frac{\partial T'}{\partial x} = -\nabla^2 V'. \]

(23)
We restrict our attention to two-dimensional disturbances (Squire’s theorem provides justification for working in two dimensions [37]) and the resulting equations are made dimensionless by scaling \((x, y, z)\) by \(h\), \(t\) by \(\varphi h^2/\kappa\), \(q'\) by \(\kappa/h\), \(T'\) by \(\beta h\), \(V'\) by \(\gamma \beta E_0 h^2\) and \(P'\) by \(\kappa \mu /hK\) to get (after discarding the primes for simplicity)

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]

\[
\frac{1}{Pr_D} \left( \frac{\partial u}{\partial t} + W_b \frac{\partial u}{\partial z} \right) = -\frac{\partial P}{\partial x} - u + Da \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{Re_D}{2} \left( \frac{\partial v}{\partial x} + T \right)
\]

\[
\frac{1}{Pr_D} \left( \frac{\partial w}{\partial t} + u Dw_b + W_b \frac{\partial w}{\partial z} \right) = -\frac{\partial P}{\partial z} - w + Da \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + R_D T
\]

\[
M \frac{\partial T}{\partial t} + W_b \frac{\partial T}{\partial z} + \frac{u}{2} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}
\]

\[
\frac{\partial T}{\partial x} = -\left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} \right)
\]

where \(D = d/dx\), \(Re_D = \gamma^2 E_0^2 \beta^2 h^2 K/\mu K\) is the AC electric Darcy-Rayleigh number, \(Pr_D = v h^2 \varphi_p^2 /K\kappa\) is the modified Darcy-Prandtl number, \(R_D = \alpha \varphi \beta h^2 K/\kappa K\) is the Darcy-Rayleigh number, \(Da = \mu_\kappa /Kh^2\) is the Darcy number, and \(M = \chi /\varphi_p\). It should be noted that the basic velocity in dimensionless form is

\[
W_b = \frac{R_D}{2} [x - \cosh(Da^{-1/2}) \sinh(Da^{-1/2}x)]
\]

Eliminating the pressure \(P\) from the momentum equations, introducing a stream function \(\psi(x, z, t)\) through

\[
u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x}
\]

and employing the normal mode analysis procedure in the form

\[
\{\psi, T, V\} = \{\Psi, \theta, \phi\}(x)e^{ia(z-c)}
\]

where \(c\) is the wave speed which is real and positive and \(a\) is the vertical wave number, we obtain

\[
\frac{1}{Pr_D} \left[ (W_b - c)(D^2 - a^2) \Psi - D^2 W_b \Psi \right] + \frac{Re_D}{2} (D\phi + \theta) = \frac{1}{ia} \left[ Da(D^2 - a^2)^2 \Psi - (D^2 - a^2)\Psi - R_D D\theta \right]
\]

\[
(W_b - Mc)\theta + \frac{1}{2} \Psi = \frac{1}{ia} (D^2 - a^2)\theta
\]

\[
D\theta + (D^2 - a^2)\phi = 0
\]

In general, \(c = c_r + ic_i\) where \(c_r\) is the phase velocity and \(c_i\) is the growth rate. It is considered that on the rigid and isothermal vertical plates the tangential electric field vanishes. Thus the associated boundary conditions are:

\[
\Psi = D\Psi = \theta = \phi = 0 \quad \text{at} \quad x = \pm 1.
\]

Eqs. (32)-(35) constitute a stability eigenvalue problem which is solved numerically by the well known Chebyshev collocation method ([Orszag [38], Canuto et al. [39]]). Accordingly, the field variables \(\Psi, \theta\) and \(\phi\) are approximated in terms of Chebyshev polynomials in the form

\[
\Psi(x) = \sum_{j=0}^{N} \xi_j(x) \Psi_j, \quad \theta(x) = \sum_{j=0}^{N} \xi_j(x) \theta_j, \quad \phi(x) = \sum_{j=0}^{N} \xi_j(x) \phi_j
\]

where \(\xi_j(x) = \cos(j \cos^{-1}x)\). The usual procedure leads to the following generalized matrix eigenvalue problem (for details see Shankar et al. [40])

\[
AX = CBX
\]

where \(c\) is the eigenvalue and \(X\) is the discrete representation of the eigenfunction; \(A\) and \(B\) are square (complex) matrices of order \(3(N+1)\). The eigenvalues and the eigenfunctions of the above matrix eigenvalue problem are determined with the aid of a QZ-algorithm which is available in the MATLAB software package in the form of built in function eig().
5. Energy norm at critical state

In order to understand the physical mechanism during the flow transition, it is necessary to define an inner-product and the associated norm. This will allow one to address the orthogonality of the eigenfunctions as well as the size of perturbations. For the temporal evolution of disturbances in an incompressible flow, the energy density of a disturbance is a physically meaningful measure of the size of the perturbation. The rate of change of two-dimensional (non dimensional) kinetic and thermal energies are given by

\[
\frac{1}{P_{TD}} \frac{\partial}{\partial t} \left( \frac{1}{2} (u^2 + w^2) \right) = - \frac{1}{P_{TD}} (uwDW_b) + R_D(wT) - \left( u^2 + w^2 \right) - Da(\nabla u)^2 + (\nabla w)^2 - \frac{R_{cwd}}{2} \left( \frac{\partial V}{\partial x} + uT \right) = E_t + E_b + E_D + E_d + E_e
\]

(38)

\[
\frac{1}{2} M \left( \frac{\partial (T^2)}{\partial t} \right) = - \frac{1}{2} (uT) - \left( (\nabla T)^2 \right) = T_c + T_d
\]

(39)

---

Table 1

Convergence of the Chebyshev collocation method.

<table>
<thead>
<tr>
<th>N</th>
<th>$R_D = 20000$, $P_{TD} = 5$, $R_{cwd} = 20$. $\sqrt{Da} = 0.5$, $a = 1 = M$</th>
<th>$R_D = 20000$, $P_{TD} = 20$, $R_{cwd} = 20$. $\sqrt{Da} = 0.5$, $a = 1 = M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-12.19739532$, $125.79237869$, $-0.00406956$, $138.33800182$</td>
<td>$-12.19739532$, $125.79237869$, $-0.00406956$, $138.33800182$</td>
</tr>
<tr>
<td>5</td>
<td>$-1.56843935$, $150.91954455$, $-8.61251762$, $161.64109986$</td>
<td>$-1.56843935$, $150.91954455$, $-8.61251762$, $161.64109986$</td>
</tr>
<tr>
<td>10</td>
<td>$-1.68402373$, $150.32443549$, $-8.83280114$, $161.66535648$</td>
<td>$-1.68402373$, $150.32443549$, $-8.83280114$, $161.66535648$</td>
</tr>
<tr>
<td>15</td>
<td>$-1.57766246$, $150.30126741$, $-7.73565882$, $161.68244144$</td>
<td>$-1.57766246$, $150.30126741$, $-7.73565882$, $161.68244144$</td>
</tr>
<tr>
<td>20</td>
<td>$-1.57746745$, $150.30253403$, $-7.73049242$, $161.62926681$</td>
<td>$-1.57746745$, $150.30253403$, $-7.73049242$, $161.62926681$</td>
</tr>
<tr>
<td>30</td>
<td>$-1.57746867$, $150.30253184$, $-7.73049791$, $161.62922666$</td>
<td>$-1.57746867$, $150.30253184$, $-7.73049791$, $161.62922666$</td>
</tr>
<tr>
<td>35</td>
<td>$-1.57746862$, $150.30253133$, $-7.73049697$, $161.62922666$</td>
<td>$-1.57746862$, $150.30253133$, $-7.73049697$, $161.62922666$</td>
</tr>
<tr>
<td>40</td>
<td>$-1.57746862$, $150.30253134$, $-7.73049698$, $161.62922666$</td>
<td>$-1.57746862$, $150.30253134$, $-7.73049698$, $161.62922666$</td>
</tr>
<tr>
<td>50</td>
<td>$-1.57746863$, $150.30253133$, $-7.73049697$, $161.62922666$</td>
<td>$-1.57746863$, $150.30253133$, $-7.73049697$, $161.62922666$</td>
</tr>
<tr>
<td>60</td>
<td>$-1.57746862$, $150.30253133$, $-7.73049698$, $161.62922666$</td>
<td>$-1.57746862$, $150.30253133$, $-7.73049698$, $161.62922666$</td>
</tr>
<tr>
<td>70</td>
<td>$-1.57746862$, $150.30253133$, $-7.73049698$, $161.62922666$</td>
<td>$-1.57746862$, $150.30253133$, $-7.73049698$, $161.62922666$</td>
</tr>
</tbody>
</table>

Table 2

Comparison of critical stability parameters.

<table>
<thead>
<tr>
<th>$\sqrt{Da} = 1$, $a = M$</th>
<th>Makinde [41]</th>
<th>Present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e$</td>
<td>$\alpha_c$</td>
<td>$\alpha_c$</td>
</tr>
<tr>
<td>0.1</td>
<td>5772.2283</td>
<td>1.02052</td>
</tr>
<tr>
<td>0.2</td>
<td>5832.2973</td>
<td>1.01989</td>
</tr>
<tr>
<td>0.3</td>
<td>6015.0334</td>
<td>1.01781</td>
</tr>
<tr>
<td>0.4</td>
<td>6328.7057</td>
<td>1.01492</td>
</tr>
<tr>
<td>0.5</td>
<td>6787.3070</td>
<td>1.01074</td>
</tr>
<tr>
<td>0.7</td>
<td>7411.1295</td>
<td>1.00561</td>
</tr>
<tr>
<td>0.8</td>
<td>8227.4284</td>
<td>0.99966</td>
</tr>
</tbody>
</table>
Table 3
Comparison of Chebyshev collocation and Galerkin method for different values of $Pr_D$ and $\sqrt{Da^{-1}}$ when $R_{ND} = 50$ and $M = 1$.

<table>
<thead>
<tr>
<th>$\sqrt{Da^{-1}}$</th>
<th>$Pr_D$</th>
<th>Chebyshev collocation method</th>
<th>Galerkin method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_D$x</td>
<td>$a_c$</td>
<td>$c_c$</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>1.374</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>25881.212807</td>
<td>0.718</td>
<td>$\pm 196.269020$</td>
</tr>
<tr>
<td>10</td>
<td>26789.475313</td>
<td>1.068</td>
<td>$\pm 207.938936$</td>
</tr>
<tr>
<td>50</td>
<td>55356.729218</td>
<td>1.274</td>
<td>$\pm 466.562448$</td>
</tr>
<tr>
<td>100</td>
<td>79604.651063</td>
<td>1.313</td>
<td>$\pm 647.255100$</td>
</tr>
<tr>
<td>200</td>
<td>115498.702807</td>
<td>1.323</td>
<td>$\pm 944.319679$</td>
</tr>
<tr>
<td>1.0</td>
<td>1</td>
<td>1.363</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>6040.636461</td>
<td>1.342</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>12102.045597</td>
<td>1.342</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>8161.211361</td>
<td>1.109</td>
<td>$\pm 238.188553$</td>
</tr>
<tr>
<td>100</td>
<td>11065.889915</td>
<td>1.206</td>
<td>$\pm 328.681413$</td>
</tr>
<tr>
<td>200</td>
<td>15656.609942</td>
<td>1.274</td>
<td>$\pm 470.376116$</td>
</tr>
<tr>
<td>4.0</td>
<td>1</td>
<td>0.934</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>254.972191</td>
<td>0.807</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>523.163845</td>
<td>0.780</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>2731.021507</td>
<td>0.792</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>5441.506959</td>
<td>0.792</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>2091.925568</td>
<td>0.718</td>
<td>$\pm 407.317435$</td>
</tr>
</tbody>
</table>

where the brackets $()$ represent the volumetric average over the volume of disturbance waves. In Eq. (38), the first term on the right hand side, $E_v$ represents the gain (loss) of the disturbance kinetic energy from (to) the mean flow through Reynolds stress, and is referred to the shear production (destruction). The term denoted by $E_b$ is due to non-isothermal effects and represents the production (destruction) of disturbance kinetic energy through the buoyancy effect. The third, fourth and fifth terms ($E_D$, $E_q$ and $E_p$) represent, respectively, the dissipation of disturbance kinetic energy due to surface drag, viscous force and electric force. The first term in $E_v$ has cropped up due to non-uniform variation in the electric field and the second term is occurred due to dielectrophoretic forces caused by the variation in dielectric constant with the non-homogeneous temperature gradient in the bulk flow. In Eq. (39), quantities termed $T_e$ and $T_q$ are associated with disturbance thermal energy due to thermal convection and disturbance thermal energy due to diffusion effect. All derivatives are approximated by fourth-order finite difference in the given domain except at boundaries. Integrals are calculated using fourth-order Simpson’s extended rule.

6. Results and discussion

The stability of natural convection in a vertical dielectric fluid saturated Brinkman porous layer under a uniform horizontal AC electric field is investigated numerically using the Chebyshev collocation method. The non-dimensional parameters involved in the present study are the Darcy-Rayleigh number $R_D$, AC electric Darcy-Rayleigh number $R_{ND}$, Darcy number $Da$, Darcy-Prandtl number $Pr_D$, and the non-dimensional group $M$. Convergence of the results is tested for different sets of parametric values by varying the order of base polynomial $N$ (see Table 1). From the table it is evident that four digits point accuracy can be achieved by retaining 31 terms in Eq. (36). As the number of terms increased in Eq. (36), the results are found to remain consistent and the accuracy improved up to 5 digits for $N=30$ and up to 7 digits for $N=50$. The results presented here are for $N=50$. The results obtained in the absence of AC electric field are compared with those of isothermal porous channel flows as the numerical results are lacking for non-isothermal case in the open literature. The results so obtained for different values of Darcy number with proper adjustment of parameters are compared with those of Makinde [41] in Table 2 and note that there is a good agreement between them. The result corresponds to the nonporous domain case ($Da^{-1}=0$) is in conformity with the one obtained by Orszag [38]. Moreover, the eigenvalue problem is also solved numerically using the Galerkin method with Legendre polynomials as trial functions

$$\Psi(x) = \sum_{n=0}^{N} a_n (1-x^2)^2 P_n(x), \quad \theta(x) = \sum_{n=0}^{N} b_n (1-x^2) P_n(x), \quad \phi(x) = \sum_{n=0}^{N} c_n (1-x^2) P_n(x)$$

where $P_n(x)$ is the Legendre polynomial of degree $n$ and $a_n$, $b_n$ and $c_n$ are constants (Singer et al. [42], Chen and Chung [43]). The results obtained for a representative set of parametric values considering 55 terms in the Galerkin expansion are shown in Table 3 along with those obtained from the Chebyshev collocation method. It is seen that the results complement with each other.

The basic natural convection flow, stationary and parallel, is not influenced by the AC electric field. Nonetheless, the Darcy number and the Darcy-Rayleigh number influence the base flow significantly. Fig. 2(a) and (b) show the influence of $Da^{-1}$ (with $R_D=2000$) and $R_D$ (with $Da^{-1}=1$) on the basic velocity $W_p$, respectively. These figures indicate that the velocity
profiles are anti-symmetric about the vertical line at \( x=0 \) but they are not precisely centro-symmetric about \( x=\pm 1/2 \). In other words, in the half region, the basic velocity is in one direction and in the other half it is in the opposite direction and it is zero at \( x=0 \). Moreover, decrease in \( Da^{-1} \) is to suppress the fluid flow (Fig. 2(a)) and a similar trend is noticed with decreasing \( R_D \) (Fig. 2(b)). Also, at higher values of \( Da^{-1} \) we recover the Darcy flow case with a linear profile, but with thin boundary layers near the two vertical walls in order to satisfy the no-slip condition.

It is inquisitive to look at the results obtained under the limiting cases of \( Re_D=0 \) (absence of electric field), \( Pr_D \to \infty \) (absence of inertial effects) and \( Da=0 \) (Darcy case) as this problem was originally discussed analytically by Gill [4].
Fig. 3. Variation of growth rate $c_i$ with wave number $a$.

The overall trend displayed in Fig. 3(a) for this case shows the stability of the basic flow for all considered values of $R_D$ which corroborate the results of Gill [4]. The basic flow is found to be stable even with the inclusion of Brinkman term and AC electric field when $Pr_D \to \infty$, and the results shown in Fig. 3(b)–(d) demonstrate that the equilibrium is always stable. Nonetheless, the study evidences an unstable behavior ($c_i > 0$) once the inertial effects are taken into consideration and this fact is evident from Fig. 3(e)–(h). A closer look at these figures reveals that there exists a threshold value of $Da^{-1}$, $R_D$
and $R_{edD}$ above which (Fig. 3(e)–(g)), and the value of $Pr_D$ below which (Fig. 3(h)) instability occurs. Moreover, the threshold values are interdependent on the parametric values involved therein.

Fig. 4(a)–(c) exhibit neutral stability curves in the $(R_D,a)$–plane for various values of $Pr_D$ (with $Da^{-1}=16$, $R_{edD}=50$, $Da^{-1}$ (with $Pr_D=10$, $R_{edD}=50$) and $R_{edD}$ (with $Pr_D=10$, $Da^{-1}=9$), respectively. The neutral stability curves exhibit single but different minimum with respect to the wave number for various values of $Pr_D$, $Da^{-1}$ and $R_{edD}$. Considered. The stability to linear perturbations corresponds to the region below and outside the neutral stability curves while the instability corresponds to the region above as well as inside each neutral stability curve. For the considered parametric values, Fig. 4(a) illustrates that there exists a threshold value of $Pr_D(=136.9)$ below which the instability always occurs as stationary mode while beyond this threshold value the instability first occurs as a travelling-wave mode and switches over to stationary mode with further increase in the value of wave number. Despite the change of instability with increasing wave number, the travelling-wave mode turns out to be the preferred mode of instability. Moreover, it is seen that $Pr_D$ exhibits a dual behavior on the stability of the basic flow depending on the mode of instability. Increasing $Da^{-1}$ is to decrease the region of stability (Fig. 4(b)) and similar is the case with increasing $R_{edD}$ (Fig. 4(c)).

The variation of $R_{dc}$ and $a_c$ as a function of $Pr_D$ is illustrated in Fig. 5(a) and (b), respectively for different values of $Da^{-1}$ when $R_{edD}=50$ and $M=1$. Fig. 5(a) clearly indicates the existence of a threshold value of $Pr_D$ at which the instability switches over from stationary to travelling-wave mode. Moreover, the threshold value of $Pr_D$ increases significantly with increasing $Da^{-1}$. The effect of increasing $Da^{-1}$ is to reinforce instability on the system. In other words, increasing $Da^{-1}$ is to destabilize the basic flow if the instability is via stationary mode. This may be attributed to the fact that decreasing Darcy number is equivalent to decrease in the ratio of viscosities, $\mu_c/\mu$ and hence its effect is to destabilize the system due to decrease in the viscous diffusion. However, when the instability is via travelling-wave mode the observed scenario is quite different. For this case, $R_{dc}$ passes through a minimum with increasing $Pr_D$ up to a certain value of $Da^{-1}$ and this trend eventually goes on diminishing with increasing $Da^{-1}$ ($= 4, 9, 16$) and leading to instability of the system with $Pr_D$. The vertical lines represent the discontinuous changes in the critical wave number $a_c$ due to the transition from stationary to travelling-wave mode (Fig. 5(b)). The figure indicates that the dependence of $a_c$ on $Pr_D$ is weak at stationary mode, whereas at travelling-wave mode $a_c$ depends strongly on $Pr_D$. Also, the effect of increasing $Da^{-1}$ is to enlarge the size of convection cells at both stationary and travelling-wave modes. The results of travelling-wave mode instability summarized in Fig. 5(c) indeed confirm the above observed behavior more evidently, which shows the variation of positive $c_c$ with $Pr_D$ for various values of $Da^{-1}$. The discontinuous changes in $c_c$ due to the transition from stationary ($c_c=0$) to travelling-wave ($c_c \neq 0$) mode are represented by the vertical lines. From Fig. 5(c) it is observed that for values of $Da^{-1}=0.04, 0.25$ and $1$, $c_c$ values pass through a minimum with increasing $Pr_D$. This trend goes on diminishing with increasing $Da^{-1}$ and for values of $Da^{-1}=4, 9$ and $16$ the critical wave speed decreases with increasing $Pr_D$.

Fig. 6 shows the instability map on the $(Pr_D, Da^{-1})$–plane showing the division of the plane into zones of stationary and travelling wave modes. It is evident that the threshold value of the modified Darcy-Prandtl number at which the transition from stationary mode to travelling-wave mode occurs increases significantly with $Da^{-1}$.

The above observed facts are also viewed and analyzed through streamlines and isotherms at the critical state. In all the contour plots, dashed lines denote negative values whereas solid lines stand for positive values. In addition, for stream function, solid and dashed lines are associated with a clockwise and counter-clockwise rotation, respectively. Figs. 7 and 8 exhibit streamlines and isotherms before and after the transition mode as a function of $Pr_D$ for $Da^{-1}=1-M$ and $R_{edD}=50$. For $Pr_D=12.1$, the flow pattern appears to be as stationary and uni-cellular. Further, a closer look at the stream line contour shows that uni-cellular pattern moving alternatively clockwise and anti-clockwise along the $z$ direction (Fig. 7(a)).
isotherms are uni-cellular and concentrate in the vicinity of the hot wall and which is evident from Fig. 8(a). The flow pattern and flow strength changes qualitatively as well as quantitatively as the mode of instability changes from stationary to travelling-wave with increasing $Pr_D$. In other words, the instability switches over from stationary to travelling-wave mode once the value of $Pr_D$ exceeds the value 12.1. When $Pr_D = 12.3$, it is observed that the convective motion is constructed by two parts, one is the vigorous cell in the right half of the porous layer adjacent to the right surface, the other is the weak circulation which is exterior to the vigorous convection cell in streamlines (Fig. 7(b)). Also, shapes of the isotherms change (Fig. 8(b)). It is further seen that the actual wavelengths are substantially larger in both streamlines and isotherms and at this stage $\Psi_{\text{max}}$ increases from 0.62 to 1.05, also $\theta_{\text{max}}$ changes from 0.017 to 0.004. This fact is also evident from Figs. 7(b) and 8(b). Further increase in $Pr_D$ is to decrease the wavelength as well as to increase the flow strength slightly (Fig. 7(c) and (d)) and also to strengthen the isotherms (Fig. 8(c) and (d)). From these figures, it is noted that the streamlines move closer
Fig. 5. Variation of (a) critical Darcy–Rayleigh number $R_{Dc}$, (b) critical wave number $a_c$ and (c) critical wave speed $c_c$ with the Darcy–Prandtl number $Pr_D$ for a fixed value of $Re_D = 50$, $M = 1$ and for various values of $\sqrt{Da}^{-1}$. (——) Stationary modes, (-----) travelling-wave modes.
and appear to be parallel at the center of the vertical fluid-saturated porous layer and bi-cellular oblate triangles occupy the whole thickness of the vertical porous layer in isotherms. With increasing $Pr_D$ (136.7 and 136.9), it is observed that the magnitude of secondary flow changes in both streamlines and isotherms (Figs. 7(e), (f) and 8(e), (f)).

To know the impact of Darcy number on the stability characteristics of the system, the results are presented for $Da^{-1} = 4$ in Fig. 9. The streamlines and isotherms are shown in Fig. 9(a, b) and (c, d), respectively, for $Pr_D = 44.2$ and 44.4. For $Pr_D = 44.2$, the pattern appears to be stationary uncellular both in streamlines and isotherms (Fig. 9(a) and (c)). As $Pr_D$ increases to 44.4 (Fig. 9(b)), a sudden change in the magnitude of secondary flow is observed (i.e., $\Psi_{\text{max}} = 0.63$ to 1.11) and convective cells transform from unicellular to bi-cellular in streamlines. In the case of isotherms, as mode changes from stationary to travelling-wave, the magnitude of isothermal lines reduce drastically ($\theta_{\text{max}} = 0.02$ to 0.003). Also, shapes of the isotherms change from unicellular to multi-cellular (Fig. 9(d)). This behavior of the streamlines and isotherms may be due to the change in the mode of instability from stationary to travelling-wave mode.

To get a clear understanding about the characteristics of neutral stability of fluid flow for a given set of parameters, energy spectrums of the secondary flow are analyzed in detail. Theoretically at the critical points, $\partial E_{\text{kin}}/\partial t = 0$ and $\partial E_{\text{dhw}}/\partial t = 0$. In other words, total kinetic energy ($E_{\text{kin}}$) as well as thermal energy ($E_{\text{dhw}}$) production must balance the energy dissipation. In all the numerical results produced here for energy analysis, this phenomenon is observed at the critical points.

The neutral stability curves of the given flow show that the threshold value of $Pr_D$, where the secondary flow changes its mode from stationary to travelling-wave mode, is a strong function of $Da^{-1}$. For understanding this phenomenon, variation of kinetic and thermal energy components are analyzed as a function of $Pr_D$ and shown in Figs. 10–12 for three different values of $Da^{-1} = 0.25, 1$ and 4, respectively, for a fixed value of $Re_{\text{tip}} = 50$ and $M = 1$. It can be seen that for $Da^{-1} = 0.25$, shear production ($E_s$) has a tendency to destabilize the flow and viscous dissipation ($E_d$) stabilizes it, in the domain [0.1, 3] of $Pr_D$. However, the contribution of energy production due to buoyancy term ($E_b$) is not negligible in this domain. Quantitatively, in this domain contribution of shear and buoyant term is around 80 and 20% approximately in the instability of flow, respectively. Furthermore, the production of kinetic energy due to shear term diminishes drastically as $Pr_D$ changes from 3.0 to 3.1, which could be associated to the change of stationary to travelling-wave mode of secondary flow. After this threshold value of $Pr_D (= 3)$, $E_s$ is responsible to stabilize the flow and buoyant instability ($E_b$) is the only instability mode. For this range of $Pr_D(> 3.1)$, other components of kinetic energy have negligible influence on the stability or instability of the flow. However, a sudden change in the role of components of kinetic energy is observed for the value of $Pr_D$ at which the mode of instability changes. Another important observation here is that the total kinetic energy production is almost balanced by energy dissipation due to viscous force term ($E_d$) in the entire domain of modified Darcy–Prandtl number (Fig. 10a). From Figs. 11(a) and 12(a), it can be concluded that, for $Da^{-1} = 1$ and 4 also kinetic energy spectrums show a similar character-

![Fig. 5. Continued](image-url)
Fig. 6. Variation of $Pr_{DT}$ (transition Darcy–Prandtl number at the critical domains) as a function of $\sqrt{Da^{-1}}$ when $Re_D = 50$, $M = 1$.

Fig. 7. The disturbance streamlines at $\sqrt{Da^{-1}} = 1 = M$ and $Re_D = 50$ for different values of $Pr_D$: 

(a) $Pr_D = 12.1$, $\Psi_{max} = 0.62$, 
(b) $Pr_D = 12.3$, $\Psi_{max} = 1.05$, 
(c) $Pr_D = 44.2$, $\Psi_{max} = 1.21$, 
(d) $Pr_D = 44.4$, $\Psi_{max} = 1.23$, 
(e) $Pr_D = 136.7$, $\Psi_{max} = 0.98$, 
(f) $Pr_D = 136.9$, $\Psi_{max} = 0.89$. 

Fig. 7. The disturbance streamlines at $\sqrt{Da^{-1}} = 1 = M$ and $Re_D = 50$ for different values of $Pr_D$. 
istics as noted above and after the threshold value of $Pr_D$. Thermal energy spectrums show that disturbance thermal energy due to thermal convection ($T_c$) has a destabilizing effect on the flow and disturbance thermal energy due to diffusion effect ($T_d$) has a stabilizing effect throughout the domain of $Pr_D$ (Figs. 10(b), 11(b) and 12(b)) for all three values of $Da^{-1}$ considered here. As can be seen from Table 4, similar to the results of Su and Chung [44], kinetic disturbances lose their energy to the mean flow by the shear destruction ($E_4$) when $Pr_D$ crosses the threshold value for all the three values of $Da^{-1}$. However, the characteristics of $E_5$ is a function of $Pr_D$ and energy is gained from the mean flow when $Pr_D$ is less than the threshold value for a given set of parameters.

**Table 4**
Variation of $R_{b}$ and energy spectrum as a function of $Pr_D$ and $\sqrt{Da^{-1}}$ at $R_{a0} = 50$ and $M = 1$.

<table>
<thead>
<tr>
<th>$\sqrt{Da^{-1}}$</th>
<th>$Pr_D$</th>
<th>$R_{b}$</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_3$</th>
<th>$E_4$</th>
<th>$E_5$</th>
<th>$E_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.0</td>
<td>49804.572790</td>
<td>0.784256</td>
<td>−0.014264</td>
<td>1.000000</td>
<td>0.230173</td>
<td>0.000096</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.1</td>
<td>51466.316779</td>
<td>0.783093</td>
<td>−0.014249</td>
<td>1.000000</td>
<td>0.231324</td>
<td>0.000093</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td>42787.29588</td>
<td>0.014915</td>
<td>−0.021932</td>
<td>0.993746</td>
<td>1.000000</td>
<td>0.000073</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>26789.475313</td>
<td>0.011729</td>
<td>−0.023020</td>
<td>0.965386</td>
<td>1.000000</td>
<td>0.000167</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.0</td>
<td>35368.893697</td>
<td>−0.006657</td>
<td>−0.023314</td>
<td>0.970099</td>
<td>1.000000</td>
<td>0.000091</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>55356.729218</td>
<td>−0.004804</td>
<td>−0.022999</td>
<td>0.972232</td>
<td>1.000000</td>
<td>0.000045</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>3.0</td>
<td>3614.991416</td>
<td>0.865652</td>
<td>−0.058229</td>
<td>1.000000</td>
<td>0.192367</td>
<td>0.001455</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>12102.04559</td>
<td>0.806516</td>
<td>−0.055751</td>
<td>1.000000</td>
<td>0.250012</td>
<td>0.000410</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.2</td>
<td>14769.039233</td>
<td>0.978545</td>
<td>−0.05364</td>
<td>1.000000</td>
<td>0.257663</td>
<td>0.000333</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.3</td>
<td>12927.360435</td>
<td>0.007679</td>
<td>−0.077779</td>
<td>0.929928</td>
<td>1.000000</td>
<td>0.000220</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.0</td>
<td>69770.095337</td>
<td>−0.013651</td>
<td>−0.082561</td>
<td>−0.904309</td>
<td>1.000000</td>
<td>0.000691</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>8161.211361</td>
<td>−0.01871</td>
<td>−0.085378</td>
<td>−0.903139</td>
<td>1.000000</td>
<td>0.000493</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>390.343298</td>
<td>1.000000</td>
<td>−0.209133</td>
<td>−0.947373</td>
<td>0.142590</td>
<td>0.015321</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>1312.208594</td>
<td>0.937712</td>
<td>−0.211236</td>
<td>−1.000000</td>
<td>0.270466</td>
<td>0.004415</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.0</td>
<td>2642.093549</td>
<td>0.886753</td>
<td>−0.204194</td>
<td>−1.000000</td>
<td>0.316662</td>
<td>0.002093</td>
<td></td>
</tr>
<tr>
<td></td>
<td>44.3</td>
<td>5871.486993</td>
<td>0.838016</td>
<td>−0.196468</td>
<td>−1.000000</td>
<td>0.358812</td>
<td>0.000896</td>
<td></td>
</tr>
<tr>
<td></td>
<td>44.4</td>
<td>5741.526922</td>
<td>−0.021873</td>
<td>−0.231796</td>
<td>−0.746564</td>
<td>1.000000</td>
<td>0.000389</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>3628.722121</td>
<td>−0.026603</td>
<td>−0.237509</td>
<td>−0.736670</td>
<td>1.000000</td>
<td>0.000937</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 8.** The disturbance isotherms at $\sqrt{Da^{-1}} = 1 = M$ and $R_{a0} = 50$ for different values of $Pr_D$. 
The variation of $R_{dc}$ and $a_c$ with $Pr_D$ is shown in Fig. 13(a) and (b) for a fixed value of $Da^{-1} = 16$ with $M = 1$ and for different values of $Re_{ad}$. For a fixed value of $Re_{ad}$, it is observed that $R_{dc}$ increases linearly with $Pr_D$ till $Pr_D < 136.8$ and exceeding which $R_{dc}$ decreases monotonically. Although the effect of increasing $Re_{ad}$ is to instill instability on the system but its influence is found to be not so significant (Fig. 13(a) and Table 5). That is, higher the AC electric field strength the less stable the system due to an increase in the destabilizing electrostatic energy to the system. If the disturbances are
Fig. 11. The rate of change of (a) kinetic energy and (b) thermal energy as a function of $Pr_D$ for $\sqrt{Da^{-1}} = 1 = M$ and $Re_D = 50$.

Fig. 12. The rate of change of (a) kinetic energy and (b) thermal energy as a function of $Pr_D$ for $\sqrt{Da^{-1}} = 2$, $M = 1$ and $Re_D = 50$.

Table 5
 Variation of $R_{dc}$ as a function of $Re_D$ and $Pr_D$ for $\sqrt{Da^{-1}} = 4$ and $M = 1$.

<table>
<thead>
<tr>
<th>$Pr_D$</th>
<th>$Re_D = 0$</th>
<th>$Re_D = 50$</th>
<th>$Re_D = 70$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_{dc}$</td>
<td>$R_{dc}$</td>
<td>$R_{dc}$</td>
</tr>
<tr>
<td>1</td>
<td>45.738381</td>
<td>30.694852</td>
<td>21.341258</td>
</tr>
<tr>
<td>5</td>
<td>265.299459</td>
<td>254.972191</td>
<td>250.596912</td>
</tr>
<tr>
<td>10</td>
<td>533.833851</td>
<td>523.163845</td>
<td>518.760672</td>
</tr>
<tr>
<td>20</td>
<td>1084.409239</td>
<td>1074.239751</td>
<td>1070.11057</td>
</tr>
<tr>
<td>50</td>
<td>2722.912919</td>
<td>2713.021507</td>
<td>2709.043272</td>
</tr>
<tr>
<td>100</td>
<td>5451.337417</td>
<td>5441.506959</td>
<td>5437.561758</td>
</tr>
<tr>
<td>136.8</td>
<td>7455.751171</td>
<td>7445.955952</td>
<td>7442.029549</td>
</tr>
<tr>
<td>136.9</td>
<td>7474.693963</td>
<td>7468.556048</td>
<td>7463.486613</td>
</tr>
<tr>
<td>150</td>
<td>3918.709473</td>
<td>3912.694057</td>
<td>3910.936924</td>
</tr>
</tbody>
</table>
Fig. 13. Variation of (a) critical Darcy–Rayleigh number $R_{Dc}$, (b) critical wave number $a_c$, and (c) critical wave speed $c_c$ with the Darcy–Prandtl number $Pr_D$ for a fixed value of $\sqrt{Da} = 4$, $M = 1$ and for various values of $Re_D$. (——) Stationary modes, (⋯⋯) travelling-wave modes.
Fig. 13. Continued

Fig. 14. Variation of $Pr_{Dc}$ (transition Darcy–Prandtl number at the critical domains) as a function of $R_{edD}$ when $\sqrt{\alpha_{cD}^{-1}} = 4, M = 1$. 
Fig. 15. The disturbance streamlines at $R_{\text{eff}} = 0$, $M = 1$ and $\sqrt{D_{\text{a}}^{-1}} = 4$ for different values of $Pr_D$.

Fig. 16. The disturbance isotherms at $R_{\text{eff}} = 0$, $M = 1$ and $\sqrt{D_{\text{a}}^{-1}} = 4$ for different values of $Pr_D$. 
stationary, dependence of the critical wave number $a_c$ on $Pr_D$ is very weak while $a_c$ at travelling-wave mode is an increasing function of $Pr_D$. The vertical line represents the discontinuous changes in $a_c$ due to transition from stationary to travelling-wave mode (Fig. 13(b)) and this behavior is unaltered with increasing $Re_{ad}$. Fascinatingly, the value of $Pr_D$ at which transition from stationary to travelling-wave instability occurs remain invariant for all the values of $Re_{ad}$ considered. This aspect has been clearly seen in Fig. 14. Additional information regarding the nature of the travelling-wave mode instability can be obtained from Fig. 13(c), which shows the variation of positive $c_c$ with $Pr_D$ for various values of $Re_{ad}$ when $Da^{-1} = 16$ and $M = 1$. In this figure, the vertical lines represent the discontinuous changes in $c_c$ due to the transition from stationary to travelling-wave mode. Also, it is observed that $c_c$ for the travelling-wave mode is a monotonically decreasing function of $Pr_D$. The variation of $Re_{ad}$ on $c_c$ is found to be not so significant.

As in the previous case, the streamlines and isotherms at the critical state for both stationary and travelling-wave modes are analyzed for this case as well. Figs. 15 and 16 show the results for different values of $Pr_D$ and for a fixed value of $Re_{ad} = 0$ and $Da^{-1} = 16$. From Figs. 15(a) and 16(a) it can be seen that for $Pr_D = 10$, the flow pattern appears to be as stationary unicellular in streamlines (Fig. 15(a)) and a flurry of activity is noticed in isotherms i.e., cells of various patterns and shapes are seen throughout the spread of porous layer (Fig. 16(a)). When $Pr_D = 50, 100$ and $136.7$, it is observed that the convective motion is uni-cellular (Fig. 15(b)-(d)) but the corresponding isotherms reveal that the undulation observed in Fig. 16(b) and (c) seem concentrated towards the hot wall (Fig. 16(b))/cold wall (Fig. 16(c)) followed by relatively milder formation at the cold wall (Fig. 16(b))/hot wall (Fig. 16(c)). Fig. 16(d) portrays a case of vigorous convection in the cell formation towards the colder end, followed by a lull towards the warmer side. The instability switches over from stationary to travelling-wave mode once the value of $Pr_D$ exceeds 136.7. When $Pr_D = 136.9$, convective cells change from uni-cellular to bi-cellular in streamlines (Fig. 15(e)). Also, shapes of the isotherms appear to be multi-cellular and distributed throughout the porous layer being sparser at the middle and getting denser towards the ends (Fig. 16(e)). It is further seen that the actual wavelengths are substantially larger in both streamlines and isotherms and at this stage, $\Psi_{max}$ increases from 0.71 to 1.09 and $\vartheta_{max}$ decreases from 0.021 to 0.002. This fact is evident from Figs. 15(e) and 16(e). It is observed that qualitatively and quantitatively the flow strength remains almost the same in both streamlines and isotherms as $Pr_D$ increases to 150 (Figs. 15(f) and 16(f)). Interestingly, secondary flow behavior remains invariant for all the values of $Re_{ad}$ considered. The change in streamlines and isotherms patterns is found to remain unaltered with increasing value of $Re_{ad}$. This fact is evident from Fig. 17 in which the results are presented for $Re_{ad} = 50$.

The energy analysis at the critical state has also been carried out as explained previously. For the above case, the threshold value of $Pr_D$ at which the change of mode takes place is 136.8. To understand the characteristics of neural stability,
Fig. 18. The rate of change of (a) kinetic and (b) thermal energy as a function of $Pr_D$ for $Re_D = 0$, $M = 1$ and $\sqrt{Da^{-1}} = 4$.

Fig. 19. The rate of change of (a) kinetic and (b) thermal energy as a function of $Pr_D$ for $Re_D = 50$, $M = 1$ and $\sqrt{Da^{-1}} = 4$.

kinetic energy spectrums are analyzed as a function of $Pr_D$ for three different values of $Re_D = 0$, 50 and 70 (see Figs. 18–20). From the figures it is evident that the energy spectrums show same behavior as a function of $Pr_D$ for all the values of $Re_D$. It is also seen that the kinetic energy spectrum profile is independent of the value of $Re_D$ and show the same characteristics before and after the threshold value of $Pr_D$ ($= 136.8$) as discussed earlier.

7. Conclusions

The linear instability of natural convection in a vertical dielectric fluid saturated Brinkman porous layer under the influence of a uniform horizontal AC electric field is investigated. Coupled stability equations are solved numerically using the Chebyshev collocation method. The effect of Darcy number, Darcy–Prandtl number and AC electric Darcy-Rayleigh number on the instability characteristics of the system is delineated. The basic stationary and parallel flow is not influenced by the AC electric field while the Darcy number alters it significantly. Despite the presence of AC electric field, the basic flow is found to be stable in the absence of inertia. Nonetheless, the inclusion of inertia causes instability in the system and the
value of modified Darcy–Prandtl number at which the transition occurs from the stationary to travelling-wave mode instability increases with increasing Darcy number but remains invariant with varying AC electric Darcy–Rayleigh number. If the instability is via stationary mode the effect of Darcy number is found to be destabilizing while it exhibits a dual behavior if the instability is via travelling-wave mode. The streamlines and isotherms presented mimic the behavior of stability curves observed before and after the change of mode of instability. The dissipation of disturbance kinetic energy due to surface drag, viscous force and non-uniform electric field show no significant contribution on the stability of the flow. But the disturbance thermal energy due to convection and diffusion shows destabilizing and stabilizing tendencies, respectively. More importantly, the production of kinetic energy due to shear term diminishes drastically as the Darcy–Prandtl number crosses its threshold value, which could be associated to the change of stationary to travelling-wave mode of secondary flow. Also the kinetic energy production is almost balanced by energy dissipation due to viscous term in the entire domain of Darcy–Prandtl number.

Acknowledgments

We thank the anonymous reviewers for their helpful suggestions which helped in improving the paper considerably. One of the authors B.M.S wishes to thank the authorities of his University for their encouragement and support.

References