Communications in Statistics - Theory and Methods

Publication details, including instructions for authors and subscription information:
<http://www.tandfonline.com/loi/lsta20>

A generalization of midzuno-sen sampling scheme for finite populations

E. Vasantha kumar ^a & T. Srivenkataramana ^b

^a Department of statistics, University of Agricultural sciences G.K.V.K., Bangalore, 560 065, India

^b Department of statistics, Bangalore University, Bangalore, 560 056, India
Published online: 27 Jun 2007.

To cite this article: E. Vasantha kumar & T. Srivenkataramana (1994) A generalization of midzuno-sen sampling scheme for finite populations, Communications in Statistics - Theory and Methods, 23:9, 2541-2559, DOI: [10.1080/03610929408831402](https://doi.org/10.1080/03610929408831402)

To link to this article: <http://dx.doi.org/10.1080/03610929408831402>

PLEASE SCROLL DOWN FOR ARTICLE

Taylor & Francis makes every effort to ensure the accuracy of all the information (the "Content") contained in the publications on our platform. However, Taylor & Francis, our agents, and our licensors make no representations or warranties whatsoever as to the accuracy, completeness, or suitability for any purpose of the Content. Any opinions and views expressed in this publication are the opinions and views of the authors, and are not the views of or endorsed by Taylor & Francis. The accuracy of the Content should not be relied upon and should be independently verified with primary sources of information. Taylor and Francis shall not be liable for any losses, actions, claims, proceedings, demands, costs, expenses, damages, and other liabilities whatsoever or howsoever caused arising directly or indirectly in connection with, in relation to or arising out of the use of the Content.

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden. Terms & Conditions of access and use can be found at <http://www.tandfonline.com/page/terms-and-conditions>

A GENERALIZATION OF MIDZUNO-SEN SAMPLING
SCHEME FOR FINITE POPULATIONS

E. Vasantha kumar

Department of statistics
University of Agricultural sciences
G.K.V.K., Bangalore, India 560 065

T. Srivenkataramana

Department of statistics
Bangalore University
Bangalore, India 560 056

Key Words and Phrases : Horvitz-Thompson estimator; IPPS scheme; location shift; Midzuno-Sen sampling scheme; systematic sampling; Yates-Grundy variance estimator.

ABSTRACT

This paper proposes a generalization of the classical Midzuno-Sen sampling procedure to allow more than one PPSWOR selection followed by SRSWOR draws. The Horvitz-Thompson (H-T) estimator, its variance and

2541

Copyright © 1994 by Marcel Dekker, Inc.

variance estimator are discussed. Two methods of controlling the sampling variability of the H-T estimator are outlined. These require simple transformations of the study or the auxiliary variates. They can also be viewed as attempts to make the sampling scheme IPPS. Simple numerical illustrations are given.

1. INTRODUCTION

The Midzuno-Sen scheme provides one of the simplest ways of making the ratio estimator exactly unbiased. However when used with the Horvitz-Thompson (H-T) estimator it is not quite efficient by itself. Efforts have been made to revise the initial probabilities so that the scheme has inclusion probabilities proportional to the size (IPPS) of units (e.g. see Rao 1963, Asok and Sukhatme 1978). It is noted that such attempts lead to stringent restrictions on the initial probabilities which are difficult to meet in practice (Brewer and Hanif 1983, p.25; Konijn 1973, p.250). As a result the use of H-T estimator following Midzuno-Sen sampling is often not preferred. Prasad and Srivenkataramana (1980) have proposed a location shift for making the Midzuno-Sen scheme an IPPS procedure. This does not impose any condition on the initial probabilities. The present paper first generalizes the Midzuno-Sen scheme to have k ($1 < k < n$) PPSWOR (probability proportional to size without replacement) selections

followed by SRSWOR (simple random sampling without replacement) draws from the depleted population. The first and second order inclusion probabilities are evaluated. The nonnegativity of the Yates-Grundy (Y-G) variance estimator is examined. Reduction of the sampling variance is attempted through revision of the selection probabilities and location shift for the study variate under the generalized sampling scheme.

2. THE MIDZUNO-SEN SCHEME

Let $\{U_1, U_2, \dots, U_N\}$ be the labelled units of a finite population of N units and let the study variate y and a related auxiliary variate x take real values (y_i, x_i) on the unit U_i , $i = 1, 2, \dots, N$. The x_i values are supposed to be known and to be positive for all the units. The objective is to estimate the population total $Y = \sum_{i=1}^N y_i$ using a random sample of n units from the population. The Midzuno-Sen (1952) scheme is to draw one unit with probabilities $p_i = x_i/X$, $X = \sum_{i=1}^N x_i$, and the other $(n-1)$ units by SRSWOR from the $(N-1)$ units remaining in the population. The H-T estimator of Y is

$$\hat{Y} = \sum_{i=1}^n y_i / \pi_i \quad (2.1)$$

with variance

$$V(\hat{Y}_{HT}) = \sum' (\pi_i \pi_j - \pi_{ij})(y_i / \pi_i - y_j / \pi_j)^2, \quad (2.2)$$

where Σ' denotes the sum over all pairs of units in the population and π_i , π_{ij} are the first and second order inclusion probabilities given by

$$\pi_i = \left[p_i (N-n) + (n-1) \right] (N-1)^{-1} \quad (2.3)$$

and

$$\pi_{ij} = \left[(p_i + p_j) (N-n) + (n-2) \right] (n-1) / (N-1)(N-2). \quad (2.4)$$

It is noted that the Y-G variance estimator

$$\hat{V}(\hat{Y}_{HT}) = S' (\pi_i \pi_j - \pi_{ij}) (y_i / \pi_i - y_j / \pi_j)^2 / \pi_{ij} \quad (2.5)$$

is nonnegative since

$$\begin{aligned} \pi_i \pi_j - \pi_{ij} &= \left[p_i p_j (N-n) + (1-p_i-p_j)(n-1) / (N-2) \right] (N-n) / (N-1)^2 \\ &\geq 0, \quad \forall i = 1, 2, \dots, N. \end{aligned} \quad (2.6)$$

In view of using the H-T estimator for Y it is desirable to have near proportionality between y_i and π_i . From (2.3) it follows that $\pi_i = np_i$, that is, the scheme becomes IPPS, when the revised probabilities are

$$p_i^* = \left[p_i n(N-1) - (n-1) \right] (N-n)^{-1}. \quad (2.7)$$

However, since p_i^* is to be positive, the initial probabilities are to satisfy the condition

$$p_i > (n-1) / n(N-1), \quad \forall i = 1, 2, \dots, N, \quad (2.8)$$

which is seen to be a rather stringent condition to be met in practice.

Deshpande (1982) and Deshpande and Ajgaonkar (1987) have discussed a variant of the Midzuno-Sen

scheme. Rao (1961) extended the Midzuno-Sen scheme to have $k=2$ PPSWOR draws and $(n-2)$ SRSWOR selections. Chaudhuri (1975) examines this scheme for making it an IPPS procedure using revised probabilities.

3. A GENERALIZATION OF THE MIDZUNO-SEN SCHEME

Consider the selection of $k(1 < k < n)$ units according to a PPSWOR scheme from the population and $(n-k)$ units from the $(N-k)$ remaining units in the population by SRSWOR. Call this scheme A. Sen (1955) has examined this scheme for obtaining unbiased estimates of between component of total error using a sample of primary stage units. Let $\pi_1(n)$ and $\pi_{1j}(n)$ denote the first and second order inclusion probabilities for the units under scheme A. Then it is easily shown that

$$\pi_1(n) = \left[\pi_1(k)(N-n) + (n-k) \right] / (N-k) \quad (3.1)$$

where $\pi_1(k)$ is the first order inclusion probability for U_1 in the first k draws. The overall second order inclusion probability is computed by noting that U_1 and U_j can get selected into the sample in the following mutually exclusive ways :

- (i) both U_1 and U_j are selected in the first k draws,
 - (ii) U_1 is selected in the first k draws and U_j in the last $(n-k)$ draws or vice versa, and
 - (iii) both U_1 and U_j get selected in the last $(n-k)$ draws.
-

This leads to

$$\begin{aligned}\pi_{ij}(n) = & \pi_{ij}(k) + \left[\pi_i(k) + \pi_j(k) - 2\pi_{ij}(k) \right] (n-k)/(N-k) \\ & + \left[1 - \pi_i(k) - \pi_j(k) + \pi_{ij}(k) \right] (n-k)(n-k-1)/(N-k)(N-k-1)\end{aligned}\quad (3.2)$$

where $\pi_{ij}(k)$ is the second order inclusion probability in the first k draws. The three components on the right side of (3.2) in fact respectively represent the probabilities of the mutually exclusive events (i), (ii), and (iii). Rearrangement of the terms in (3.2) leads to

$$\begin{aligned}\pi_{ij}(n) = & \pi_{ij}(k) \left[1 + (n-k)(n-k-1)/(N-k)(N-k-1) - 2(n-k)/(N-k) \right] \\ & + \left[\pi_i(k) + \pi_j(k) \right] (n-k)(N-n)/(N-k)(N-k-1) \\ & + (n-k)(n-k-1)/(N-k)(N-k-1)\end{aligned}\quad (3.3)$$

In this set up we consider the H-T estimator (2.1) with variance as in (2.2) where π_i and π_{ij} are now read as $\pi_i(n)$ and $\pi_{ij}(n)$. When $k=2$ the scheme is the same as that discussed in Rao (1961), where the inclusion probabilities are expressed in terms of the initial probabilities p_i . For the same case Seth (1966) has given alternative expressions for the inclusion probabilities and with $k=2$, (3.1) and (3.2) reduce to these expressions. It is easily seen that when $k=1$, scheme A reduces to the classical Midzuno-Sen scheme and expressions for $\pi_i(n)$ and $\pi_{ij}(n)$ reduce to those in (2.3) and (2.4).

4. VARIANCE ESTIMATION

Under scheme A any PPSWOR sampling method can be used for the first k draws. When $k=1$, the procedure is identical with the classical Midzuno-Sen scheme for which the Y-G variance estimator is known to be nonnegative. For $k=2$, Rao (1961) has proved this property by expressing the inclusion probabilities in terms of the initial probabilities. Seth (1966) has shown the same result through an alternative expression for the overall second order inclusion probabilities while Lanke (1974) establishes this result through an argument involving probabilities of exclusion.

Consider in general the Y-G variance estimator under scheme A. This is given by (2.5) with π_i and π_{ij} replaced by $\pi_i(n)$ and $\pi_{ij}(n)$. To examine the nonnegativity of this estimator we express the difference $A_{ij}(n) = [\pi_i(n)\pi_j(n) - \pi_{ij}(n)]$ in terms of $B_{ij}(k) = [\pi_i(k)\pi_j(k) - \pi_{ij}(k)]$. Relevant substitutions from (3.1) and (3.3) in $A_{ij}(n)$ and rearrangement of the terms give

$$A_{ij}(n) = B_{ij}(k) (N-n)^2 / (N-k)^2 + \left[1 - \pi_i(k) - \pi_j(k) + \pi_{ij}(k) \right] (N-n)(n-k) / (N-k)^2 (N-k-1),$$

which reduces to the expression given by Seth (1966) when $k=2$. An alternative expression for $A_{ij}(n)$

convenient for further discussion is

$$A_{ij}(n) = \left[B_{ij}(k)(N-n-1)(N-k) + (1-\pi_i(k))(1-\pi_j(k))(n-k) \right] \left[(N-n)/(N-k)^2(N-k-1) \right] \quad (4.1)$$

Thus $A_{ij}(n) \geq 0$ whenever $B_{ij}(k) \geq 0$. This condition is known to hold for all i and j when $k=1$ or 2 as mentioned above.

For $2 < k < n$, the nonnegativity of $A_{ij}(n)$ is not assured in general. Then one may consider the modified variance estimators suggested in the literature (e.g. Rao, 1988) or use for the first k draws rejective sampling which has been shown to have nonnegative $B_{ij}(k)$ (Hájék 1981, p.75, 92; Lanke 1974, Sampford 1967) so that under scheme A the variance estimator is nonnegative.

For $k > 2$ one may examine the sufficient condition for the nonnegativity of the Y-G variance estimator by using the following approximate expression to $O(N^{-2})$ (Hartley and Rao, 1962):

$$\pi_{ij}(k) = k^{-1}(k-1)\pi_i(k)\pi_j(k) \left[1 + (\pi_i(k) + \pi_j(k))/k - \sum_{t=1}^N \pi_t(k)^2/k^2 \right]. \quad (4.2)$$

Using this we obtain

$$B_{ij}(k) = \pi_i(k)\pi_j(k)k^{-2} \left[k - (\pi_i(k) + \pi_j(k)) - \sum_{t=1}^N \pi_t(k)^2/k \right] (k-1). \quad (4.3)$$

This leads to the condition

$$\pi_i(k) + \pi_j(k) \leq \sum_{t=1}^N \pi_t(k)^2/k + k/(k-1) \quad (4.4)$$

in order that $B_{ij}(k) \geq 0$, for all i and j .

For schemes where $\pi_i(k) = kp_i$, for example as in systematic sampling, (4.4) reduces to

$$p_i + p_j \leq \sum_{t=1}^N p_t^2 + 1/(k-1), \quad \forall i, j. \quad (4.5)$$

This is not a very stringent condition. For a given set of p_i 's and k , this can be verified for its validity. Alternatively it may be used to fix an upper limit on k , the number of WOR draws as follows. Let p_0 be the largest p_i . Then (4.5) is satisfied if

$$k \leq 1 + 1/(2p_0 - \sum_{t=1}^N p_t^2) \quad (4.6)$$

Under scheme A, it is of specific interest to consider PPS systematic sampling for the first k draws in view of its operational convenience. One main drawback with systematic sampling is that the joint inclusion probabilities needed for variance estimation are zero for some pairs. But this difficulty disappears here even though $\pi_{ij}(k)=0$ for some pairs, as seen from (3.3). This may be considered as an advantage of another sampling scheme following systematic sampling.

5. VARIANCE REDUCTION UNDER SCHEME A

The sampling variance of the H-T estimator depends on the proportionality between y_i and $\pi_i(n)$ which can be

improved by (i) revision of initial probabilities p_i or (ii) transforming the y -variate. Each of these approaches is illustrated below.

(i) Revision of p_i

Let p_i^* denote the revised probabilities which make $\pi_i(n) = np_i$. For the first k draws let us choose a scheme which has $\pi_i(k)$ of the form kp_i^* . In view of the relation (3.1) we now get

$$np_i = \left[kp_i^*(N-n) + (n-k) \right] / (N-k)$$

which gives

$$p_i^* = \left[p_i n(N-k) - (n-k) \right] / k(N-n) \quad (5.1)$$

Since p_i^* is to be positive for all i , we require

$$\min p_i = p' > (n-k)/n(N-k). \quad (5.2)$$

For $k=1$, (5.2) reduces to the condition (2.8) for the Midzuno-Sen scheme to be IPPS which is known to be very stringent and not satisfied by most of the real-life populations. For low values of k the condition (5.2) appears difficult to be satisfied in practice. As k increases the stringency of the condition disappears as seen from the following illustration.

The condition (5.2) was verified for four natural populations (Raj 1965, Hanurav 1967, Sukhatme 1954) each

TABLE I

Values of $\phi = (n-k)/n(N-k)$

Population	p'	k	1	2	3	4	5
		ϕ	.043	.037	.029	.021	.011
A	.023					x	
B	.026					x	
C	.014						x
D	.016						x

'x' mark indicates the least value of k
for which condition (5.2) is met.

with N=20. A sample of size 6 was assumed. The results are summarized in Table I.

It is noted that the value of k may have to be quite close to n in order that (5.2) is met when p' is some what low. In such cases the number of SRSWOR draws will be small. And use of a scheme like systematic sampling for the first k draws under scheme A retains simplicity of the selections. However condition (4.5) or

(4.6) concerning the nonnegativity of the Y-G variance estimator is to be now satisfied by the revised probabilities p_i^* . For populations A, B, C and D this is verified to be true.

The condition (5.2) can also be expressed as

$$k > n \left[1 - (N-n)p'/(1-np') \right] \quad (5.2a)$$

A practical implication of this is that one can always choose a $k < n$ such that scheme A becomes IPPS.

(ii) Transformation of y-variate

The revision of p_i , if they are based on x_i , in effect transforms the size measure x_i . Rao (1988) has considered the transformation of the auxiliary variate for Midzuno-Sen sampling scheme. Alternatively one may examine a location shift for y to reduce the variance of the H-T estimator under scheme A. This could be particularly relevant in a survey with several study variates. Consider the case where

$$y_i = \alpha + \beta x_i \quad (5.3)$$

and the k WOR draws are made such that $\pi_i(k) = c + dx_i$,
 $\forall i = 1, 2, \dots, N$. Then using (3.1) we get

$$\pi_i(n) = \gamma + \delta x_i,$$

where $\gamma = \left[c(N-n) + (n-k) \right] / (N-k)$ and

$\delta = d(N-n)/(N-k)$. Even here $y_i/\pi_i(n)$ is not a constant. Whereas

$$\begin{aligned}
 \left[y_i - (\alpha - \beta\gamma/\delta) \right] / \pi_i(n) &= (\beta x_i + \beta\gamma/\delta) / (\gamma + \delta x_i) \\
 &= \beta(x_i + \gamma/\delta) / \delta(x_i + \gamma/\delta) \\
 &= \beta/\delta
 \end{aligned}$$

is a constant for all $i = 1, 2, \dots, N$.

Given the scheme for the k WOR draws, the values of c and d and hence those of γ and δ will be known. Thus it follows that if a good assessment of α and β can be made even as late as at the estimation stage in a survey, the sampling variance can be controlled by making a transformation

$$z_i = y_i - \theta, \quad i = 1, 2, \dots, N, \quad (5.4)$$

where $\theta = a - b\gamma/\delta$, and a , b are the assessed values of α and β respectively. Then, under scheme A, we consider the unbiased estimator

$$\hat{y}_{HT}^* = \sum_{i=1}^n \left[z_i / \pi_i(n) \right] + N\theta \quad (5.5)$$

as an alternative to $\hat{y}_{HT}$. The sampling variance of $\hat{y}_{HT}^*$ is given by (2.4) with z_i now replacing y_i .

The parameter θ needed for making the transformation (5.4) may be obtained by assessing α and β (Method 1) or by estimating θ (Method 2) which minimizes the sampling variance of $\hat{y}_{HT}^*$ as discussed in Srivenkataramana and Tracy (1986) and Stuart (1986). Under Method 1, we may use the scatter diagram of

$y_i/\pi_i(n)$ plotted against $x_i/\pi_i(n)$ and assess the intercept on the vertical axis and also the slope of the best fitting line to be used as a and b . Method 2 leads to

$$\theta_{\text{opt}} = \frac{\sum' A_{ij}(n) \left[y_i/\pi_i(n) - y_j/\pi_j(n) \right] \left[1/\pi_i(n) - 1/\pi_j(n) \right]}{\sum' A_{ij}(n) \left[1/\pi_i(n) - 1/\pi_j(n) \right]^2}$$

$$= \frac{\tau_1}{\Delta_1} \quad (\text{say})$$

in which case

$$V(\hat{y}_{\text{HT}}^*) = V(\hat{y}_{\text{HT}}) - \tau_1^2 / \Delta_1 \quad (5.6)$$

Thus the reduction in sampling variance due to the optimum location-shift for y is τ_1^2 / Δ_1 . In practice, θ_{opt} may be estimated by

$$\hat{\theta} = \frac{S' A_{ij}(n) \left[y_i/\pi_i(n) - y_j/\pi_j(n) \right] \left[1/\pi_i(n) - 1/\pi_j(n) \right] / \pi_{ij}(n)}{S' A_{ij}(n) \left[1/\pi_i(n) - 1/\pi_j(n) \right]^2 / \pi_{ij}(n)}$$

Since $\pi_{ij}(n) > 0 \quad \forall \quad i, j$ under scheme A , $\hat{\theta}$ is unbiased for θ_{opt} . The impact of using $\hat{\theta}$ in the place of θ_{opt} on sampling variance has been examined in Stuart (1986).

Method 1 is motivated by the linear relation (5.3) between y and x which is well justified when x and y are highly correlated, x being a measure of size. Method 2

does not presume this relation and arrives at variance-minimizing θ directly.

6. CONCLUSION

The proposed sampling scheme A provides a generalization of the Midzuno-Sen scheme and it is more amenable for being made IPPS unlike the Midzuno-Sen scheme. There is also a particular advantage that the second order inclusion probabilities are positive for all pairs of units to allow unbiased estimation of the sampling variance. In particular one can conveniently use PPS systematic sampling for the first k draws. The sufficient condition for the nonnegativity of the Y-G variance estimator which is known to hold for $k=1$ and 2, is satisfied under scheme A provided the initial probabilities meet a condition, which is not very restrictive. Alternatively one can place an upper limit on the value of k for this condition to be satisfied.

The revision of initial probabilities for variance reduction imposes conditions on them which become less restrictive as k increases. PPS systematic sampling seems to be particularly applicable for the first k draws in such a situation. The location shift for the study variate discussed for reducing the sampling variability of the estimator can be carried out provided the intercept and slope of the population regression

line can be assessed or a certain parameter θ can be estimated from the sample. It is noted that this is not difficult to accomplish. The reduction in sampling variance can be estimated from the sample itself.

An usual difficulty with unequal probability sampling schemes is that they are complex for application for larger sample sizes. There is also the associated problem of nonnegative variance estimators or the second-order inclusion probability vanishing for certain pairs. Such schemes are sometimes not amenable for being modified to become nearly IPPS in order to control the sampling variance. In this context the proposed scheme A provides a practical and useful method of sampling with unequal probabilities.

ACKNOWLEDGEMENT

The authors thank Prof. T.J. Rao for some useful discussion.

BIBLIOGRAPHY

- Asok, C. and Sukhatme, B.V. (1978), A note on Midzuno scheme of sampling, an abstract, J. Indian Soc. Agric. Statist, 30(2), 131.
- Brewer, K.R.W. and Hanif, M. (1983), *Sampling With Unequal Probabilities*, Lecture Notes in Statistics, Vol. 15, Springer-Verlag, New York.

- Chaudhuri, A. (1975), A simple method of sampling without replacement with inclusion probabilities exactly proportional to size, *Metrika*, 22, 147-152.
- Deshpande, M.N. (1982), A new fixed size sampling procedure with unequal probabilities of selection, *J. Indian Soc. Agric. Statist*, 34, 71-75.
- Deshpande, M.N. and Ajgaonkar, S.G.P. (1987), A generalization of Midzuno's sampling procedure, *Aust. J. Statist*, 29(2), 188-192.
- Hajék, J. (1981), *Sampling From A Finite Population*, Marcel Dekker, Inc., New York.
- Hanurav, T.V. (1967), Optimum utilization of auxiliary information: π ps sampling of two units from a stratum, *J. Roy. Statist. Soc., Ser B*, 29(2), P.386.
- Hartley, H.O. and Rao, J.N.K. (1962), Sampling with unequal probabilities and without replacement, *Ann. Math. Statist.*, 33, 350-374.
- Konijn, H.S. (1973), *Statistical Theory of Survey Design and Analysis*, North-Holland Publishing Company, Amsterdam.
- Lanke, J. (1974), On nonnegative variance estimators in survey sampling, *Sankhya*, Ser C, 36, 33-42.
- Midzuno, (1952), On the sampling systems with probability proportional to sum of sizes, *Ann. Inst. Statist. Math.*, 3, 99-107.
-

- Prasad, N.G.N. and Srivenkataramana, T. (1980), A modification to the Horvitz-Thompson estimator under the Midzuno-Sen sampling scheme, *Biometrika*, 67, 709-711.
- Raj, D. (1965), Variance estimation in randomized systematic sampling with probability proportional to size, *J. Amer. Statist. Assoc.* 60, 282.
- Rao, J.N.K. (1961), On the estimate of the variance in unequal probability sampling, *Ann. Inst. Statist. Math.*, 13(1), 57-60.
- Rao, J.N.K. (1963), On two systems of unequal probability of sampling without replacement, *Ann. Inst. Statist. Math.*, 15, 67-72.
- Rao, J.N.K. (1988), Variance estimation in sample surveys, *Handbook of Statistics, Vol. 6*, eds. P.R. Krishnaiah and C.R. Rao, North-Holland, 427-447.
- Rao, T.J. (1988), Transformation on the auxiliary variate for Midzuno-Sen sampling scheme, *J. Indian Soc. Agric. Statist.*, 40(3), 173-177.
- Sampford, M.R. (1967), On sampling without replacement with unequal probabilities of selection, *Biometrika*, 54, 499-513.
- Sen, A.R. (1952), Present status of probability sampling and its use in estimation of farm characteristics, an abstract, *Econometrica*, 27, 130.
- Sen, A.R. (1955), On the selection of n primary sampling

units from a stratum structure ($n \geq 2$), Ann. Math. Statist., 26, 744-751.

Seth, G.R. (1966), On estimators of variance of estimation of population total in varying probabilities, J. Indian Soc. Agric. Statist., 18(2), 52-56.

Srivenkataramana, T. and Tracy, D.S. (1986), Transformations after sampling, Statistics, 17, 597-608.

Stuart, A. (1986), Location-shifts in sampling with unequal probabilities, J. Roy. Statist. Soc. Ser A, 149, 349-365.

Sukhatme, P.V. (1954), *Sampling Theory Of Surveys With Applications*, Ames, Iowa : Iowa State College Press, 279-280, circles 1-20 and 21-40.

Received September, 1993.