External Bremsstrahlung of $^{147}$Pm in PbCl$_2$ and CdO

H.C. Manjunatha *, K.C. Suresh, B. Rudraswamy

Department of Physics, Bangalore University, Bangalore 560056, Karnataka, India

1. Introduction

Sommerfeld [1] was first to work out the theory of External Bremsstrahlung (EB) for non-relativistic electrons. Bethe and Heitler [2] developed theory for relativistic electrons using Born approximation. Tseng and Pratt [3] have made EB cross section calculations based on the description of the atom as a static spherically symmetric charge distribution of infinite mass, using four different central potentials, viz., point coulomb, Thomas–Fermi, modified Thomas–Fermi and modified Hartree–Fock–Slater by using a partial wave expansion procedure. Seltzer and Berger [4] extended Tseng–Pratt theory for EB produced not only in the field of atomic nucleus but also in the field of an atomic electron and evaluated EB cross section data for various elements. The theories discussed above give cross-sections which are applicable to thin target EB spectra. However, Bethe and Heitler [2] has given an expression for EB produced in thick target elements. Dhaliwal et al. [5] measured EB spectra of $^{147}$Pm in thick target elements. Their experimental results showed better agreement with Tseng and Pratt theory for medium and high-Z elements than Bethe–Heitler theory. In our earlier work [6], a study of EB spectra excited by beta particle of $^{204}$Tl in thick target compounds has been reported. Tajinder Singh et al. [7] measured Bremsstrahlung spectra produced by beta particles of the $^{204}$Tl in thick target elements of Al, Ti, Sn and Pb at photon energies from 5 keV to 10 keV and compared with theory. Lixia Tian et al. [8] measured Bremsstrahlung spectra generated by 5–25 keV electron impact on thick targets elements and the experimental data are compared with the simulation results of X-ray spectra obtained from the general-purpose Monte Carlo code PENEOPE. EB imaging technique [9] is being used worldwide to diagnosis of metastases of a human bone (treated as a compound) in which the given beta source is placed. Most of the EB works of beta have been carried out using only metal as a thick target but using compound as a thick target is lacking [10]. Markowicz and VanGriken [11] proposed a new expression for the prediction of the Bremsstrahlung intensity $I$ for compounds to take into account the self absorption of Bremsstrahlung and electron back scattering and to obtain the accurate description of the Bremsstrahlung process

$$I = \text{con} \left( \frac{AE_i}{E_g} \right) Z_{\text{mod}}(E_0 - E_i)(1 - f)$$

(1)

Here,

$$Z_{\text{mod}} = \frac{\sum Z_i w_i A_i}{\sum Z_i w_i}$$

(2)

$E_i$ and $E_0$ are emitted photon energy and incident electron energy, respectively. $I$ indicates the number of elements in the compound and $W_i, A_i, Z_i$ are the weight fraction, atomic weight and atomic number of the $i$th element present in the compound respectively. $f$ is a function of $E_0$, $E_i$ and composition. For pure elements $f=0$. The object of present study is to compare the experimental EB spectra from thick target compound like lead chloride (PbCl$_2$) and cadmium oxide (CdO) excited by beta particles of $^{147}$Pm with Tseng and Pratt theory. Thus, the present measurements are expected to check the validity of Tseng and Pratt theory for compound thick targets.

2. Evaluation of theoretical EB spectrum

In the present work, we have evaluated $Z_{\text{mod}}$ using Markowicz’s Eq. (2). The estimated $Z_{\text{mod}}$ for PbCl$_2$ and CdO are 73.48 and 42.29, respectively. We have evaluated the EB cross-section for these compounds using Lagrange’s interpolation technique, Seltzer’s theoretical EB cross section data given for

* Corresponding author.
E-mail address: manjunathhc@rediffmail.com (H.C. Manjunatha).
elements and the evaluated results of $Z_{\text{mod}}$ using the following expression:

$$\sigma_{\text{eval}} = \sum \left( \prod_{j=1}^{Z_{\text{mod}}} \frac{Z_{j}}{Z_{j} + Z_{0}} \right) \sigma_{z}$$

where lower case $z$ is the atomic number of the element of known EB cross-section $\sigma_{z}$ adjacent to the modified atomic number ($Z_{\text{mod}}$) of the compound whose EB cross-section $\sigma_{\text{eval}}$ is desired and upper case $Z$ are atomic numbers of other elements of known EB cross-section adjacent to $Z_{\text{mod}}$. The number $n(T,k)$ of EB photons of energy $k$ when all of the incident electron energy $T$ completely absorbed in thick target is given by Bethe and Heitler\cite{2} is

$$n(T,k) = N \int_{1+k}^{T} \frac{\sigma(E,k)}{\left( -\frac{dE}{dx} \right) E} dE$$

where $\sigma(E,k)$ is EB cross-section at photon energy $k$ and electron energy $E$, $N$ is the number of atoms per unit volume of target and $E$ is the energy of an electron available for an interaction with the nucleus of the thick target after it undergoes a loss of energy per unit length ($-\frac{dE}{dx}$). For a beta emitter with end point energy $T_{\text{max}}$, EB theoretical spectral distribution is given by

$$S(k) = \frac{\int_{T_{\text{max}}}^{T_{\text{max}}} n(T,k)p(T) dT}{\int_{T_{\text{max}}}^{T_{\text{max}}} p(T) dT}$$

where $p(T)$ is the beta spectrum of $^{147}\text{Pm}$. Evaluated results of $\sigma(E,k)$ of (3), tabulated values of $(-\frac{dE}{dx})$ of Seltzer and Berger\cite{4}, and the $p(T)$ are used to get $S(k)$ for the target compounds.

3. Experimental details

The beta source $^{147}\text{Pm}$ ($\Delta J=0$, 225 keV) was obtained from Bhabha Atomic Research Centre, Bombay, India and the measured spectrum of this beta source is as shown in Fig. 1. The details of experimental arrangements are similar to previous work\cite{12} as shown in Fig. 2. A 3.8 cm x 3.8 cm NaI(Tl) crystal detector mounted on photomultiplier was coupled to a PC based ND-62 sophisticated 16k multi-channel analyzer (MCA). The crystal was housed in a hallow lead chamber. The lead chamber was lined with aluminum in side. Target compounds such as PbCl$_2$ and CdO in the fine powder form were filled in Perspex planchet of 1 cm diameter. The thickness of these compounds was so chosen to stop all the beta particles. The various properties such as density and melting point are checked with usual simple methods and found are constant during experiment. Beta source was placed in a Perspex stand at a distance of 12.5 cm above the face of the detector. The target compound was placed between the detector and the source. The geometry was carefully adjusted to see that crystal was fully exposed to the EB emitted from the target. The MCA was calibrated using various gamma sources like $^{170}\text{Tm}$ (84 keV), $^{57}\text{Co}$ (122 keV), $^{201}\text{Hg}$ (279 keV), $^{51}\text{Cr}$ (320 keV) and $^{137}\text{Cs}$ (662 keV) of energies ranging between 84 keV and 662 keV before and after experiment to check the stability of the instrument. A Perspex sheet with thickness sufficient to stop all beta particles is placed on the top of the target compound and with source in position, the spectrum EB+IB+BG was taken. Here IB and BG are internal Bremsstrahlung and background, respectively. The Perspex was then placed below the target compound and the
spectrum IB+BG was recorded for the same time. The difference in the two spectra gives raw EB spectrum. Data were accumulated each time for 12 hr. Among the several sets of data recorded, the averages of the six sets of consistent data were used for the final analysis.

4. Results and discussions

The observed pulse height distribution is the original photon spectrum folded by the response function of the detector system. Hence observed pulse height distribution has been unfolded using the method of Lidden starfelt [13]. The experimental EB pulse height distribution was corrected for energy resolution of the detector. This correction varied from 5.5% at 50 keV and decreased with increase in photon energy due to better resolution of detector at higher energies. The correction due to escape of iodine K X-rays was also calculated. It was significant only for low energy photons in the EB spectrum due to their low penetration and high photoelectric cross-section. The correction for the loss of iodine K X-rays was 15% at 50 keV and 1.1% at 150 keV. The correction due to Compton continuum was found to be negligible. The experimental spectra were also corrected for absorption of EB photons in Aluminum container of the detector. The correction due to self absorption of EB photons in a target was applied by using the attenuation coefficients from WINXCOM program [14,15]. The spectrum is divided by beta source strength to get finally true EB spectrum which gives number of photons per $m_C^2$ per unit photon yield $S(k)$. The measured EB pulse-height distribution of $^{147}$Pm beta particles and various corrections for the PbCl$_2$ target is as shown in Fig. 3.

The unfolded measured spectra obtained for PbCl$_2$ and CdO along with the evaluated theoretical spectra are shown in Fig. 4 for the energy range 20–180 keV. Fig. 5 gives the ratio of experimentally measured spectrum to the evaluated theoretical spectrum for PbCl$_2$ and CdO. It is evident from these figures that the experimental points show closer agreement with theory in the low energy range than in the higher energy end of the spectra. The deviation remains almost same with increase of $Z_{\text{mod}}$ of compound at a given energy. The positive deviation of experimental point with theory is found to be less than 2% at 20 keV, less than 5% at 80 keV, less than 7% at 120 keV and less than 10% at 180 keV for all compounds. In conclusion, experimental results show fairly good agreement at low energy end of spectrum and some deviation (less than 10%) at higher

---

Fig. 3. Experimental EB pulse-height distributions for $^{147}$Pm beta particles and various corrections for the PbCl$_2$ target: curve I: Compton continuum, curve II: correction due to the iodine K X-ray escape peak and detector resolution, curve III: EB pulse height distribution for $^{147}$Pm in PbCl$_2$ target.

Fig. 4. Unfolded measured EB spectrum (circle) with the theoretical distribution (line) for PbCl$_2$ and CdO.

Fig. 5. The ratio of experiment/theory.
energy end of spectrum with the theory. The deviation between experiment and theory, especially at high energy regions of spectra could be understood qualitatively as follows. The thick target calculation of EB spectra assume isotropic production of Bremsstrahlung, because thick target multiple collisions may be expected to smear out the angular dependence. But single radiative collisions of electrons in which all the energy is lost still retain angular dependence especially at high energy regions of beta spectrum. The emission in the forward direction increases as the energy of the electrons increases, i.e. as we go towards the end-point energy of the beta spectrum.

The measured EB spectrum can be used to estimate Bremsstrahlung dose $D(x)$ at a distance $x$ from the source and it is given by

$$D(x) = \tau \sum A_i \phi(x)$$  \hspace{1cm} (6)

where $\phi(x)$ is the specific absorbed fraction of energy at distance $x$ from source and it is given by

$$\phi(x) = \frac{\mu_{en} \exp(\mu x) B_{en}}{4\pi x^2 \rho}$$  \hspace{1cm} (7)

Here $\mu_{en}$ is linear absorption coefficient of photons of given energy, $\mu$ is linear attenuation coefficient of photons of given energy, $B_{en}$ is absorption build up factor, $\rho$ is density of the medium, $\tau$ is the residence time of activity which is the ratio of actual activity to the administrated activity. The quantity $A_i$ numerically equal to $(2.13 n_i E_i)$, where $n_i$ is the frequency of occurrence of emissions with energy $E_i$. The quantities $n_i$ and $E_i$ are provided by measured Bremsstrahlung spectrum.

References