Non-linear study of electrohydrodynamic Rayleigh–Taylor instability in a composite fluid–porous layer

N. Rudraiah^{a,*}, Krishna B. Chavaraddi^b, I.S. Shivakumara^a

^a UGC-Centre for Advanced Studies in Fluid Mechanics, Department of Mathematics, Bangalore University, Bangalore 560001, India

^b Department of Mathematics, Government First Grade College, Yellapur (U.K.) 581359, India

ARTICLE INFO

Article history:

Received 15 May 2010

Accepted 5 April 2011

Available online 29 April 2011

Keywords:

Non-linear

Electrohydrodynamic Rayleigh–Taylor instability

Predictor–corrector numerical technique

ABSTRACT

The non-linear electrohydrodynamic RTI in presence of electric field bounded above by porous layer and below by a rigid surface, have been studied based on electrohydrodynamic approximations in the effect similar to the Stokes and lubrication approximations. The non-linear problem is studied numerically in the present paper using the Adams–Bashforth predictor and Adams–Moulton corrector numerical techniques. In the conclusion, the non-linear problem discussed here is quite different from that of Babchin et al. (1983) [10] considering the plane Couette flow. The present problem is greatly influenced by the slip velocity at the interface between porous layer and thin film. It is not amenable to analytical treatment as that of Babchin et al. [10]. Therefore, numerical solutions have to be found. Fourth-order accurate central differences are used for spatial discretization using predictor and corrector numerical technique.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

The phenomenon of instability of the interface between a heavy fluids supported by a lighter fluid known as the Rayleigh–Taylor instability (RTI) has been studied extensively starting from Rayleigh [1] and Taylor [2]. A detailed account of RTI is given by Chandrasekhar [3]. This problem has attracted considerable interest (see Refs. Sharp [4] and Mikaelian [5]) over the years because of its natural occurrence and of its importance in science, engineering and technology. It plays an important role in the areas of inertial confinement fusion; astrophysics and geophysics (see Refs. Bernstein and Book [6], Rudraiah [7]).

Although copious literature is available on linear RTI, the work on non-linear RTI is very sparse. McCrory et al. [8] have given the simulations of the RTI of ablatively accelerated thin-shell fusion targets and showed that the non-linear evolution exhibits spike amplitude saturation. A simple model is derived heuristically for the non-linear evolution of the RTI by Baker and Freeman [9]. Babchin et al. [10] have studied the non-linear saturation of RTI in thin films. They have found that the combined action of flow shear and surface tension is the essence of the saturation mechanism. Shivamoggi [11] has used the method of strained co-ordinates in investigating the non-linear RTI problem. After incorporating the corrections pointed out by Malik and Singh [12] he obtained a revised expression for the non-linear cut-off wave number, which separates the region of

stability from that of instability. Mohamed and Shehawey [13] have investigated non-linear electrohydrodynamic RTI in the absence of surface charges and a charge-free surface separating two semi-infinite dielectric fluids influenced by a normal electric field subjected to non-linear deformations. Allah and Yahia [14] have studied the non-linear RTI in the presence of magnetic field and also mass and heat transfer using the simplified formulation. Later, Varma and Shukla [15] have studied the linear and non-linear properties of Rayleigh–Taylor modes.

Rudraiah et al. [16] have studied the linear and non-linear RTI in a viscous fluid layer bounded below by a rigid surface and above by a porous layer based on the approximations in effect, which are similar to lubrication and Stokes approximations. The linear problem has been studied analytically, while the non-linear problem is studied numerically. They have shown that the stability curve can be controlled by the porous-slip parameter. A weakly non-linear but numerically tractable model has been developed for the ablative RTI with a finite bandwidth by Ikegawa and Nishihara [17].

Nevertheless, much attention has not been given in the literature on the study of non-linear RTI in a poorly conducting fluid bounded above by a porous layer and below by a rigid surface in the presence of electric field in spite of its importance in varied problems. Therefore, in this paper we have investigated non-linear RTI in a composite fluid–porous layer. The evolution of the interface is analyzed numerically by employing the fourth-order Adams–Bashforth predictor and Adams–Moulton corrector methods. The control of instability of the interface is analyzed in detail.

To achieve this objective, this paper is planned as follows. The basic equations for poorly conducting fluid in the presence of an electric field called EHD equations are given in Section 2 with

* Corresponding author.

E-mail addresses: rudraiahn@hotmail.com (N. Rudraiah), ckrishna2002@yahoo.com (K.B. Chavaraddi), shivakumarais@gmail.com (I.S. Shivakumara).

suitable approximations and boundary conditions. The dispersion relation of RTI in EHD denoted by ERTI in the presence of electric field in fluid layer bounded above by a porous layer is derived in Section 3. The importance of maintaining an electric potential either parallel to or opposing the direction of gravity in the control of ERTI growth rate is discussed in this section. The important conclusions are drawn in the final section.

2. Mathematical formulation

The physical configuration is same as the one considered in Fig. 1. It consists of a thin target shell in the form of a thin film of unperturbed thickness h (region 1) filled with an incompressible, viscous, poorly electrically conducting light fluid of density ρ_f bounded below by a rigid surface at $y=0$ and above by an incompressible, viscous poorly (Fig. 1) conducting heavy fluid of density ρ_p saturating a dense porous layer of large extent compared to the shell thickness h . Also, the electrodes are embedded at the rigid surface $y=0$ as well as at the interface $y=h$ and there by an electric field is generated in fluid–porous medium composite system. The fluid in the thin film is set in motion by acceleration normal to the interface whereas in the porous layer it is assumed to be static and small perturbations are amplified when acceleration is directed from the lighter fluid in the thin film to the heavier fluid in the porous layer. The instability at the interface in the presence of electric field is known as the electrohydrodynamic Rayleigh–Taylor instability (ERTI). To investigate this ERTI, we consider a rectangular coordinate system (x, y) with the x -axis parallel to the film and the y -axis normal to it. The interface between the porous layer and thin film (fluid) is described by $\eta(x, t)$.

Following are the basic equations for film–porous layer composite system:

The conservation of mass:

$$\nabla \cdot \vec{q} = 0. \quad (2.1)$$

The conservation of momentum:

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} = -\nabla p + \mu \nabla^2 \vec{q} + \rho_e \vec{E}. \quad (2.2)$$

The conservation of electric charges:

$$\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla) \rho_e + \nabla \cdot \vec{J} = 0. \quad (2.3)$$

Maxwell's equations:

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_e} \quad (2.4a)$$

$$\nabla \times \vec{E} = 0 \quad \text{or} \quad \vec{E} = -\nabla \phi \quad (2.4b)$$

$$\vec{J} = \sigma \vec{E} \quad (2.4c)$$

$$\sigma = \sigma_0 [1 + \alpha_h (C - C_0)] \quad (2.4d)$$

where, $\vec{q} = (u, v)$ is the fluid velocity, $\vec{E}$ the electric field, $\vec{J}$ the current density, σ the electrical conductivity, ρ_e the density of charges, ϵ_e the dielectric constant, ϕ the electric potential, p the pressure, C the concentration, σ_0 the electrical conductivity at the reference concentration C_0 , α_h is the volumetric expansion coefficient of σ , μ the fluid viscosity and ρ the fluid density.

In view of Eq. (2.4b), $\vec{E}$ can be expressed as

$$\vec{E} = -\nabla \phi \quad (2.5)$$

where ϕ is the electric potential.

The electrical conductivity σ varies with concentration C as in Eq. (2.4d). Then assuming negligible advection of concentration, we have

$$\frac{d^2 C}{dy^2} = 0 \quad (2.6)$$

with

$$C = C_0 \quad \text{at} \quad y = 0, \quad (2.7a)$$

$$C = C_1 \quad \text{at} \quad y = h. \quad (2.7b)$$

Solving Eq. (2.6) using the given conditions and substituting this solution in Eq. (2.4d), we get

$$\sigma = \sigma_0 [1 + \alpha y] \approx \sigma_0 e^{\alpha y} \quad (2.8)$$

where $\alpha = \alpha_h \Delta C / h$ and $\Delta C = C_1 - C_0$. We assume the frequency of charge distribution is smaller than the corresponding relaxation frequency of the electric field, and hence the time derivative of ρ_e is negligible compared to $\nabla(\sigma E)$ in Eq. (2.3). From this, we get

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \alpha \frac{\partial \phi}{\partial y} = 0. \quad (2.9)$$

The above equation has to be solved subject to the boundary conditions

$$\phi = v_0 \frac{x}{h} \quad \text{at} \quad y = 0 \quad (2.10a)$$

$$\phi = v_0 \frac{(x - x_0)}{h} \quad \text{at} \quad y = h \quad (2.10b)$$

where v_0 is the applied electric potential. These conditions arise due to embedded electrodes at $y=0$ and $y=h$ and permits a linear variation of ϕ with x .

In solving Eqs. (2.1)–(2.4), following Rudraiah et al. [18], we make use of the following electrohydrodynamic approximations:

- (i) The electrical conductivity of the liquid, σ , is negligibly small, that is $\sigma \ll 1$.
- (ii) The film thickness h is much smaller than the thickness H of the dense fluid above the film. That is $h \ll H$
- (iii) The surface elevation η is assumed to be small compared to film thickness h . That is $\eta \ll h$
- (iv) The Strouhal number S , a measure of the local acceleration to inertial acceleration in Eq. (2.2), is negligibly small. That is

$$S = \frac{L}{TU} \ll 1$$

Fig. 1. Physical configuration.

where $U = v/L$ is the characteristic velocity, v the kinematic viscosity, $L = \sqrt{\gamma/\delta}$ the characteristic length and $T = \mu\gamma/h^3\delta^2$ the characteristic time.

These approximations are usually called the Stokes and lubrication approximations, which help to neglect many terms, particularly the induced magnetic field and the non-linear terms in the basic equations. We also assume that the heavy fluid in the porous layer is almost static because of heavy creeping flow approximation in a densely packed porous medium, which is needed to use the Saffman [19] slip condition.

Let us non-dimensionalize the equations using

$$x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad u^* = \frac{u}{\delta h^2/\mu_f}, \quad v^* = \frac{v}{\delta h^2/\mu_f},$$

$$p^* = \frac{p}{\delta h}, \quad \rho_e^* = \frac{\rho_e h^2}{\epsilon_e v_0}, \quad E^* = \frac{Eh}{v_0} \quad (2.11)$$

become (after neglecting the asterisks for simplicity)

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad (2.12)$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + We\rho_e E_x \quad (2.13)$$

$$0 = -\frac{\partial p}{\partial y} + We\rho_e E_y \quad (2.14)$$

where

$We = \epsilon_e v_0^2/h^3\delta$ is the electric number.

Substituting Eq. (2.10) in Eq. (2.9) and in the boundary conditions (2.10), we obtain

$$\frac{\partial^2 \phi}{\partial y^2} + \alpha \frac{\partial \phi}{\partial y} = 0 \quad (2.15)$$

with boundary conditions

$$\phi = x \quad \text{at } y = 0, \quad (2.16)$$

$$\phi = x - x_0 \quad \text{at } y = 1. \quad (2.17)$$

The solution of Eq. (2.14), using the above boundary conditions, is

$$\phi = x - \frac{x_0}{1-e^{-\alpha}}(1-e^{-\alpha y}). \quad (2.18)$$

Fig. 2. Shape of interface growth rate shape of interface growth rate for a porous parameters $\beta=0.1, 2.0$ and 100.0 with fixed values of $We=0.25, B=0.02$, (from top to bottom).

Eq. (2.4a), using Eq. (2.18), becomes

$$\rho_e = -\frac{x_0 \alpha^2}{1-e^{-\alpha}} e^{-\alpha y} \quad (2.19)$$

and hence

$$\rho_e E_x = -\rho_e \frac{\partial \phi}{\partial x} = \frac{\alpha^2 x_0 e^{-\alpha y}}{(1-e^{-\alpha})}. \quad (2.20)$$

3. Dispersion relation

To find the dispersion relation, first we have to find the velocity distribution from Eq. (2.13) using the following boundary and surface conditions:

(i) The no-slip condition at the rigid surface:

$$u = 0 \quad \text{at} \quad y = 0 \quad (3.1)$$

Fig. 3. Different values of $We=0.25, 0.5, 0.75$ and 1 with fixed values of $B=0.02, \beta=2, \ell=0.75$.

(ii) The Saffman [19] slip condition:

$$\frac{\partial u}{\partial y} = -\alpha_p \sigma_p u \quad \text{at } y = 1 \quad (3.2)$$

(iii) The kinematic condition:

$$v = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad \text{at } y = 1 \quad (3.3)$$

(iv) The dynamic condition:

$$p = -(1 \pm We)\eta - \frac{1}{B} \frac{\partial^2 \eta}{\partial x^2} \quad \text{at } y = 1 \quad (3.4)$$

where $B = \delta h^2 / \gamma$ is the Bond number, $\sigma_p = h / \sqrt{k}$ is the porous parameter and $\eta = \eta(x, y, t)$ is the elevation of the interface. It may be noted here that the kinematic condition given by Eq. (3.3) is non-linear.

Fig. 4. Different values of $B=0.01, 0.02, 0.03$ and 0.04 with fixed values of $\beta=2, We=0.25, \ell=0.75$.

The solution of (2.13) subject to the above conditions is

$$u = \frac{Py^2}{2} - Wea_3 e^{-\alpha y} + a_1 y + a_2 \quad (3.5)$$

where

$$a_1 = -P \left(\frac{\beta+2}{2(\beta+1)} \right) + Wea_4$$

$$a_2 = \frac{Wea_0}{\alpha^2}$$

$$a_3 = \frac{x_0}{1-e^{-\alpha}}$$

$$a_4 = \frac{\beta(1-\alpha)e^{-\alpha}-\beta}{(\beta+1)} a_3$$

$$\beta = \alpha_p \sigma_p$$

$$P = \frac{\partial p}{\partial x}$$

After integrating Eq. (2.12) with respect to y between $y=0$ and 1 and using Eq. (3.5), we get

$$v(1) = v_1 = - \int_0^1 \frac{\partial u}{\partial x} dy = \left(\frac{4+\beta}{12(1+\beta)} \right) \frac{\partial^2 p}{\partial x^2}. \quad (3.6)$$

Then Eq. (3.3), using Eqs. (3.6) and (3.4), becomes

$$\begin{aligned} \frac{\partial \eta}{\partial t} + \frac{1}{2(1+\beta)} \left[(1 \pm We) \frac{\partial \eta}{\partial x} + \frac{1}{B} \frac{\partial^3 \eta}{\partial x^3} \right] \frac{\partial \eta}{\partial x} + We(x_0 - a_4) \\ = - \left(\frac{4+\beta}{12(1+\beta)} \right) \left[(1 \pm We) \frac{\partial^2 \eta}{\partial x^2} + \frac{1}{B} \frac{\partial^4 \eta}{\partial x^4} \right]. \end{aligned} \quad (3.7)$$

Let us analyze the interface evolution by this equation.

The process described here is quite different from a process in which the film is bounded by a fluid with moving boundaries, instead of porous layer, discussed by Babchin et al. [10]. Therefore, we use Eq. (3.7) to study the non-linear interface evolution. Eq. (3.7) is not amenable to analytical treatment and hence we solve it numerically using 4th order central differences in space and time as explained below. For time-integration of Eq. (3.7), the Adams–Bashforth predictor and Adams–Moulton corrector steps of fourth order are used, as described by Chapra and Canale [20]. Spatial derivatives are described by the following central difference formulae of fourth-order accuracy:

$$\frac{\partial \eta}{\partial x} \rightarrow \frac{1}{12\Delta x} [\eta(i-2) - 8\eta(i-1) + 8\eta(i+1) - \eta(i+2)] \quad (3.8)$$

$$\begin{aligned} \frac{\partial^2 \eta}{\partial x^2} \rightarrow \frac{1}{12\Delta x^2} [-\eta(i-2) + 16\eta(i-1) - 30\eta(i) + 16\eta(i+1) \\ - \eta(i+2)] \end{aligned} \quad (3.9)$$

$$\begin{aligned} \frac{\partial^3 \eta}{\partial x^3} \rightarrow \frac{1}{8\Delta x^3} [-\eta(i-3) - 8\eta(i-2) + 13\eta(i-1) - 13\eta(i+1) + 8\eta(i+2) \\ - \eta(i+3)] \end{aligned} \quad (3.10)$$

$$\begin{aligned} \frac{\partial^4 \eta}{\partial x^4} \rightarrow \frac{1}{6\Delta x^4} [-\eta(i-3) + 12\eta(i-2) - 39\eta(i-1) + 56\eta(i) \\ - 39\eta(i+1) + 12\eta(i+2) - \eta(i+3)]. \end{aligned} \quad (3.11)$$

Here $\eta(i-2)$ stands for the value of η at the position $x-2\Delta x$. The integer i indicates the i th grid point.

The initial condition used in the numerical integration is a sine-wave with wave number ℓ and is of the form

$$\eta(x, 0) = \eta_0 \sin(\ell x) \left(0 < x \leq \frac{\pi}{2\ell} \right). \quad (3.12)$$

Here the amplitude η_0 is assumed to be small. In our numerical computation we use $\eta_0 = 10^{-4}$ in non-dimensional form and periodic boundary conditions have been applied in the x -direction.

4. Results and discussion

The non-linear electrohydrodynamic RTI (ERTI) in a fluid layer bounded above by a porous layer and below by rigid surface is investigated. This equation is solved numerically using fourth-order differences in space and time and the results are depicted in Figs. 2–7.

In Figs. 2–4, we discuss the spatial structure of growth rate of the interface in terms of

$$n_{NL} - n_{NL, \max} = \frac{1}{\eta} \frac{\partial \eta}{\partial t} - \frac{1}{\eta_{\max}} \frac{\partial \eta_{\max}}{\partial t}$$

at an early stage (initial line) before instability occurs for wave number $\ell = 0.75$ and other parameters defined earlier. In Figs. 2–4 six waves are contained in the interval the case of small wave numbers. The peaks reflect the position of the wave modes. Figs. 5 and 6 represent whether the full numerical solutions deviate from a simple harmonic behaviour of η , namely $\eta = \eta_0 \sin(\ell x)$ for $\ell = 0.75$, where only part of the wave is shown.

Fig. 5(a)–(c) shows that the spatial structures of the interface for $t > 0$ and Fig. 2(a)–(c) describes the interface at $t = 0$. Initial

Fig. 5. Comparison between full numerical wave number of $\ell = 0.75$ and three porous parameters $\beta = 0.1, 2.0, 100.0$ with fixed values of $We = 0.25, B = 0.02$, (from top to bottom).

profile is symmetric in the interval $0 \leq x \leq 16\pi$. But for $t > 0$, we notice that the symmetry is lost, since the maximum moves farther from one of the zeros and closer to the other one. However, this process of steepening of the forward faces of the profile does not result in the breakup of the interface, because of the effects of surface tension, slip and electric parameter. Initially these parameters may have negligible effects, but as times progresses those parameters play an important role in the stabilization. In Fig. 5(a)–(c) only one part of the wave is shown as we move from top to bottom and clearly notice that the symmetry can be obtained with the effect of slip due to porous layer and hence reduced the growth rate of RTI at the interface considerably for $t > 0$.

From Fig. 6(a)–(d) it is clear that the interface profile is symmetric in the interval $0 \leq x \leq 16\pi$ for initial time $t=0$. As we move from Fig. 5(a)–(d) (top to bottom), we observe that the symmetry can be recovered for increasing the electric parameter, We . Therefore, the effect of electric field is to reduce the asymmetry of the system and hence the system makes the stable.

Also, Fig. 4(a)–(d) represents that the interface is symmetric in the interval $0 \leq x \leq 16\pi$ for $t=0$. For $t > 0$, initially symmetry can be lost as shown in Fig. 7(a) and with increase in the Bond number the symmetry can be recovered and the results are depicted in Fig. 7(b)–(d). Hence for suitable choice of the Bond number values the symmetry can be maintained at the interface and hence reduce the growth rate of RTI at the interface.

Fig. 6.

Fig. 7.

It may also be noted that the full numerical solution for all cases is not possible because of the limitation of the numerical scheme. In this problem computation of the influence for different values of We ($=0.25-1$). Also similar behaviour happens when β is very large. In addition to this full numerical solution becomes unstable for such cases.

Acknowledgement

The work supported by ISRO under the research projects nos. ISRO/RES/2/338/2007-08 and ISRO/RES/2/335/2007-08. ISRO's financial support to carry out this research is gratefully acknowledged. One of us (KBC) wishes to thank the Principal, Government First Grade College, Yellapur (U.K.) for the encouragement and support.

References

- [1] L. Rayleigh, Scientific Papers, vol. 2, Cambridge University Press, Cambridge, England, 1900, p. 200.
- [2] G.I. Taylor, The instability of liquid surfaces when accelerated in a direction perpendicular to their planes I, Proc. R. Soc. London, Ser. A 201 (1950) 532.
- [3] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Dover Publications, New York, 1961.
- [4] D.H. Sharp, An over view of Rayleigh–Taylor instability, Physica D 12 (1984) 3.
- [5] K.O. Mikaelian, Effect of viscosity on Rayleigh–Taylor and Richtmyer–Meshkov instabilities, Phys. Rev. E 47 (1993) 375.
- [6] I.B. Bernstein, D.L. Book, Effect of compressibility on the Rayleigh–Taylor instability, Phys. Fluids A 26 (2) (1983) 453.
- [7] N. Rudraiah, Effect of porous lining on reducing the growth rate of Rayleigh–Taylor instability in the inertial fusion energy target, Fusion Sci. and Technol. 43 (2003) 1.
- [8] R.L. McCrory, L. Montierth, R.L. Morse, C.P. Verdon, Nonlinear evolution of ablation-driven Rayleigh–Taylor instability, Phys. Rev. Lett. 46 (1981) 336.
- [9] L. Baker, J.R. Freeman, Heuristic model of the nonlinear Rayleigh–Taylor instability, J. Applied Phys. 52 (2) (1981) 655.
- [10] A.J. Babchin, A.L. Frenkel, B.G. Levich, G.I. Shvashinsky, Nonlinear saturation of Rayleigh–Taylor instability in thin films, Phys. Fluids 26 (1983) 3159.
- [11] B.K. Shivamoggi, Comments on nonlinear Rayleigh–Taylor instability, Astrophys. Space Sci. 97 (1) (1983) 221.
- [12] S.K. Malik, M. Singh, Non-linear Rayleigh–Taylor instability, Astrophys. Space Sci. 90 (1) (1983) 45.
- [13] A.E.M.A. Mohamed, E.S.F. El Shehawey, Nonlinear electrohydrodynamic Rayleigh–Taylor instability. II—a perpendicular filed producing surface charge, Phys. Fluids 26 (1983) 1724.
- [14] M.H.O. Allah, A.A. Yahia, Nonlinear Rayleigh–Taylor instability in the presence of magnetic field and mass and heat transfer, Astrophys. Space Sci. 181 (1991) 183.
- [15] R.K. Varma, P.K. Shukla, Linear and nonlinear Rayleigh–Taylor modes in nonuniform dusty magnetoplasmas, Phys. Scr. 51 (1995) 522.
- [16] N. Rudraiah, C. Wagner, G.S. Evans, R. Friedrich, Nonlinear study of Rayleigh–Taylor instability in thin films past a porous layer, Indian J. Pure Appl. Math. 29 (4) (1998) 417.
- [17] T. Ikegawa, K. Nishihara, Ablation on weakly nonlinear Rayleigh–Taylor instability with finite band width, Phys. Lett. 89 (11) (2002) 115001–115004.
- [18] N. Rudraiah, B.S. Krishnamurthy, R.D. Mathad, The Effect of oblique magnetic field on the surface instability of a finite conducting fluid layer, Acta Mech. 119 (1996) 165.
- [19] P.G. Saffman, On the boundary condition at the surface of a porous medium, Stud. Appl. Math. 50 (1971) 93.
- [20] S.C. Chapra, R.P. Canale, Numerical methods for Engineers, 2nd ed., McGraw Hill Pub. Corp., New York, 1998.